Shake Table Tests of a Six-Span Steel Bridge Model for Nonlinearity Identification

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ABSTRACT

Most of real structures exhibit nonlinear behaviour during their lifetime. Nonlinear behaviour of a structure can be as a result of a local effect (friction in joints, joint and link flexibility, and nonlinear contact) or a global nonlinearity (geometric nonlinearities, boundary conditions, and nonlinear material behaviours). The presence of nonlinearity in a structural system changes its dynamic characteristics, hence the use of linear techniques is improper and, in some cases, impossible for prediction of the system behaviour. In this paper, the nonlinear behaviour of steel bridges is modelled using the vibration measurements collected during a series of shake table testing. In this modelling technique, the vibration data is trained using a time series autoregressive modelling technique and a clustering algorithm to categorise the linear and nonlinear behaviour of steel bridge structures. The vibration measurements have been recorded from a six-span steel truss bridge model during shake table testing. The bridge model has been excited using various amplitudes of several ground motions. The comparison of numerical and experimental data shows that the developed modelling technique is able to model the nonlinear behaviour of the bridge model once it is subjected to various levels of excitation.

1 INTRODUCTION

It is crucially important to detect structural damages at an early stage before irreversible consequences. However, despite the extensive literature summarising damage identification techniques for Structural Health Monitoring (SHM) applications over the last years, there are still some needs for further development of SHM systems, which can provide very early identification of any alterations in a dynamic system [1]. In most of the proposed damage identification techniques, civil infrastructures are assessed in their linear range and linear-based modelling method are used for dynamic analysis and condition assessment purposes. However, it should be mentioned that most of the real civil structures show nonlinear behaviour due to several reasons. The nonlinear behaviour of a structure can be as a result of a local effect, such as friction in joints, joint and link flexibility, and nonlinear contact, or a global nonlinearity, such as geometric nonlinearities, boundary conditions, and nonlinear material behaviours. Existence of each of these nonlinearities in a system can alter its dynamic behaviour, thus the use of a linear-based approach is improper to model such dynamic systems [2]. A statistical pattern recognition paradigm is among the most effective approaches in development of SHM system. This paradigm consists of three main components, including data acquisition, feature extraction, and feature classification, which can be implemented for an automated SHM system. Several SHM feature extraction techniques are employed fitting linear models to measured system output before and after the occurrence of damage. The damage indicators are then defined using any changes in these model parameters. More recently, the physic-based modelling approach was expanded to time series regression modelling, according to which of the model parameters and residual errors are utilised as damage indicators [3,4]. In addition, there are several methods proposed for identifying the presence of nonlinearities of a system in the literature [5,6]. Some of these techniques utilised characteristics features of a nonlinear system, such as distortion of Frequency Response Function to detect the system nonlinearity. A few used the validity of principals of a linear system to identify nonlinearity in a dynamic system. However, most techniques are based on the comparison between the response of a linear system and an unknown system. Some of the researchers have suggested the use of higher order FRF's [7], while others consider spectral analysis and different Autoregressive Models, such as Nonlinear Autoregressive Moving Average with eXogenous inputs (NARMAX) models to detect the presence of nonlinearity in a dynamic system [5].

In this paper, a vibration-based nonlinearity identification technique is used to identify early changes in a dynamic system prior to any significant damages, especially after earthquakes. This technique combines vibration data sets with time-series autoregressive modelling to detect nonlinearity in a dynamic system. The technique can categorise the linear and nonlinear behaviours of a structure, when it is subjected to various levels of excitation source. In order to verify the performance of the method, a series of shaking table tests was conducted on a steel truss bridge model in the laboratory environment.

2 METHODOLOGY

In this part, the vibration-based nonlinearity identification technique is presented for analysing earthquake-induced vibration data. This method is based on linear time-series modelling and testing the validity of basic principles of the linear models. The vibration-based nonlinearity identification technique utilises the time series Autoregressive Moving Average with Exogenous Inputs (ARMAX) modelling and probability theory to categorise the linear and nonlinear dynamic behaviours of a dynamic system. The concept of this method was extracted from the fact that residual error of a linear ARMAX model follows a normal distribution.

As mentioned, there are different types of methods to model the behaviour of linear time-invariant and dependent systems. Autoregressive model is a type of random process to model the time series. Autoregressive with eXogenous Inputs (ARX) model is one form of autoregressive models, which has been frequently used by many researchers. One of the disadvantages of ARX modelling to simulate dynamic behaviour of a system is its limitation to express the noise term due to the fact that it can represent the system disturbance as a discrete error. An ARMAX model is a more complete and flexible form of an ARX model that has an additional term as Moving Average (MA) (polynomial $C(q)$) to describe the disturbance dynamics of the system [8,9]. The following equation describes the ARMAX model:

$$y(t)+\sum_{k=1}^{na} a_k\, y(t-k)=\sum_{k=1}^{nb} b_k\, x(t-nk-k+1)+\sum_{k=1}^{nc} c_k\, e(t-k)+e(t) \tag{1}$$

where a_k, b_k and c_k are the unknown coefficients of the model to be estimated and n_a, n_b, n_c and n_k are the orders of the ARMAX model. Term $e(t - k)$ is the white-noise disturbance value. Using the backshift operator (q), the above equation can be expressed as follow:

$$A(q)\ y(t) = B(q)\ x(t) + C(q)\ e(t) \tag{2}$$

where *A(q)*, *B(q)* and *C(q)* designate the AR, X and MA polynomials of the model in the delay operator q^{-1}, which can be respectively defined by:

$$A(q) = I + A_1 q^{-1} + ... + A_{na} q^{-na} \tag{3}$$

$$B(q) = B_0 + B_1 q^{-1} + ... + B_{nb} q^{-nb} \tag{4}$$

$$C(q) = I + C_1 q^{-1} + ... + C_{nc} q^{-nc} \tag{5}$$

In this study, in order to categorise the linear and nonlinear behaviour of the system, different ARMAX models are constructed from various amplitudes of earthquake-induced signals. Then, the residual errors can be estimated by the predicted output from ARMAX models, which is ideally a Gaussian process for a stationary signal [10], and the measurement output $y(t)$. These residual errors (*E*) are presented by the following equations for two groups of earthquake-induced signals.

$$E_L = y_L(t) - \bar{y}_L(t) \tag{6}$$

$$E_{un} = y_{un}(t) - \bar{y}_{un}(t) \tag{7}$$

where the subscript *L* represents a reference status that assumes the structure behaves linearly. Also, subscript *un* shows an unknown status of the structure to be categorised using the identification process. In this study, a vibration data set induced using a low-amplitude earthquake excitation has been assumed to represent the linear model of the structure as the reference model. The behaviour of the structure under other amplitudes of excitation is investigated in comparison to this reference model. The predicted responses extracted from the reference ARMAX model and the ARMAX models represented unknown status of the structure, can be expressed according to Eqs. (8) and (9):

$$\bar{y}_L = -\sum_{k=1}^{na} a_k\, y_L(t-k) + \sum_{k=1}^{nb} b_k\, x_L(t-nk-k+1) + \sum_{k=1}^{nc} c_k\, e(t-k) \tag{8}$$

$$\bar{y}_{un} = -\sum_{k=1}^{na} a_k\, y_{un}(t-k) + \sum_{k=1}^{nb} b_k\, x_{un}(t-nk-k+1) + \sum_{k=1}^{nc} c_k\, e(t-k) \tag{9}$$

Once the amplitude of excitation increases, the performance of the target structure changes from its reference state. Residual errors extracted from ARMAX time series models provide very significant information representing this change. In this study, ARMAX models are constructed using various vibration data sets and their residual errors are compared to their counterparts extracted from a reference ARMAX model. This difference will increase, as the amplitude of excitation increases, and the structure starts to behave nonlinearly from a specific threshold. This is because the linear time series ARMAX models are able to simulate only the linear models of the structure and they cannot represent the nonlinear features of the structural responses.

In this paper, the difference between residual errors generated from the reference linear model and other unknown models is investigated using Empirical cumulative distribution function (ECDF) of time series residuals. This technique is employed in order to control the similarity between the various data sets. The ECDFs of residuals are respectively presented by $\hat{G}(E_L)$ and $\hat{G}(E_{un})$ for the reference linear model and other unknown models and are estimated using the following equations:

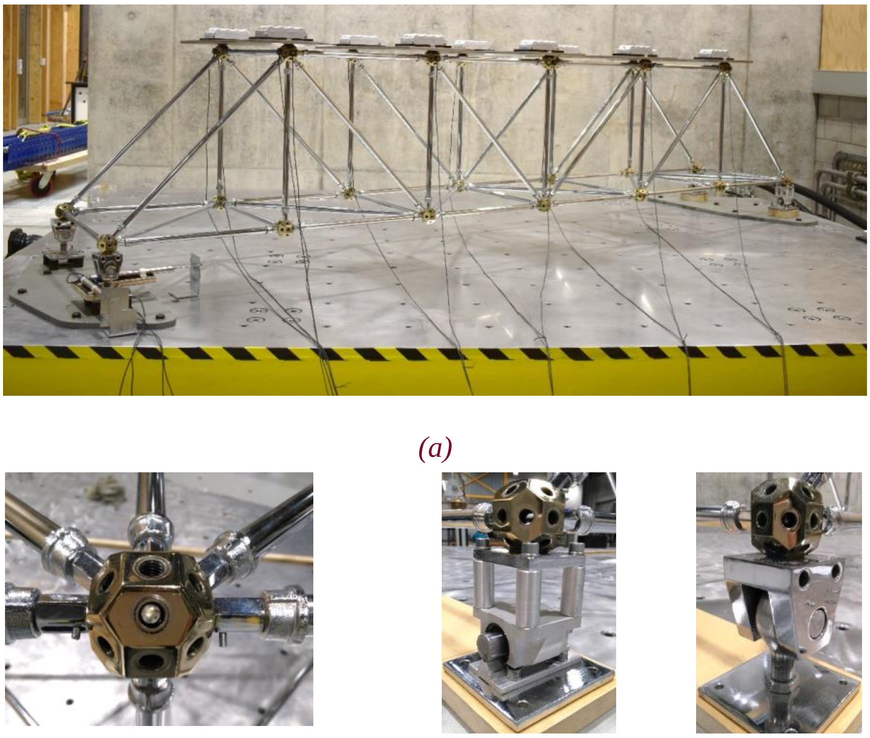
$$\hat{G}(E_L) = \frac{1}{n}\sum_{i=1}^{n} I\left(\left(E_L\right)_i \le (E_L)\right) \tag{10}$$

$$\hat{G}(E_{un}) = \frac{1}{n}\sum_{i=1}^{n} I\left(\left(E_{un}\right)_i \le (E_{un})\right) \tag{11}$$

Using the mentioned equations, the empirical cumulative distribution functions of each time series residuals are plotted to show the differences between residual errors of ARMAX models from the reference and unknown time series. In the next part, the analysis results will be presented.

3 EXPERIMENTAL TESTS

In this part, a series of shaking table test on a steel truss bridge model is presented to validate the accuracy of the proposed vibration-based nonlinearity identification technique introduced in the previous section. First, some descriptions are provided regarding the test instrumentation and setup. Then, the analysis results obtained using the nonlinearity identification algorithm are presented.



(a)

(b) *(c)*

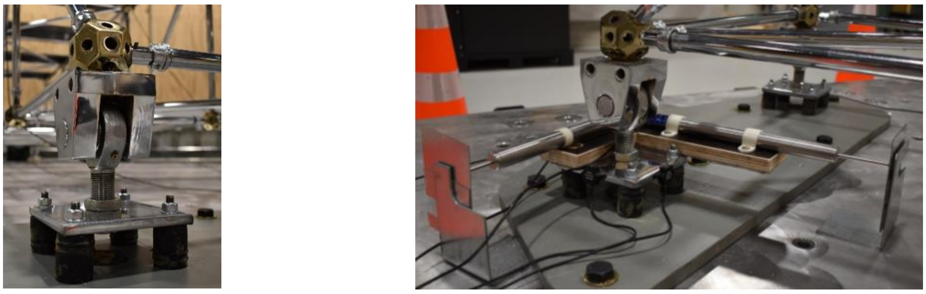
Figure 1: (a) The bridge model on the shaking table instrumented by wireless and wired accelerometers, (b) Steel ball joints, (c) roller and pinned Structural supports.

3.1 Description of shaking table tests

A six-span steel truss bridge model (as shown in Figure 1), made of MERO space frame joining system (Tube-Node system), was used as the testbed structure for these series of shaking table tests. The structural system consists of tubular steel members connected together with spherical forged steel ball joints. The tubes are connected to ball joints by means of a cone welded to the end of the tube through which a high tensile bolt is screwed into the ball by means of a sleeve. Each span of the bridge structure consists of horizontal and vertical steel tubes with the length of 60.5 cm and diagonal steel tubes with the length of 85.5 cm. The outer diameter of all the steel tubes is 2.0 cm with a thickness of 0.25 cm. In total, the six-span bridge structure has 42 of the horizontal and vertical elements, 22 of the diagonal elements, and 24 of the ball joints.

The bridge model was placed on two pinned supports at one end and two roller supports at another end. The supports were mounted on timber base plates with the thickness of 7 mm. Then, the base plates were fixed to the shaking table using several strong bolts. As is obvious, the bridge model was aligned in diagonal direction of the shake table to simulate the real behaviour of bridge structure, when subjected to earthquake excitation.

In addition, to provide high inertia mass for the bridge model during dynamic testing, five steel plates with approximate weight of 10 kg were attached to top chord of the bridge in transverse direction of the structure.



(a) *(b)*

Figure 2: (a) Rubber-based pinned support, and (b) Instrumentation setup of the support.

As mentioned, the steel bridge model was supported using two roller and two pinned supports. In order to simulate nonlinear behaviour in the bridge structure, four rubber mounts were attached to each pinned support of the bridge model. The mounts, made of natural rubber, were cylindrically shaped in diameter of 30 mm and height of 40 mm. Due to the characteristics of rubbers in shear, they were selected as the main source of nonlinearity for the bridge structure. Using these elements, different levels of nonlinearity can be simulated using various amplitudes of excitation source. To do so, the bridge model was excited using different amplitudes of ground motion to control the nonlinearity degree of rubber-based supports. Figure 2(a) shows one of the rubber-based supports.

Table 1: Characteristics of the sensors utilised throughout the dynamic tests.

Channel	Direction	Location
ACC 0	-	Shake table
ACC 1	Transverse	joint 1
ACC 2	Transverse	joint 2
ACC 3	Transverse	joint 3
ACC 4	Transverse	joint 4
ACC 5	Transverse	joint 5
ACC 6	Transverse	joint 6
ACC 7	Transverse	joint 7
ACC 8	Transverse	joint 8
ACC 9	Transverse	joint 9
ACC 10	Transverse	joint 10
ACC 11	Longitudinal	joint 6
ACC 12	Longitudinal	joint 1
ACC 13	Transverse	Rubber base
ACC 14	Longitudinal	Rubber base
LVDT 0	Transverse	Rubber base
LVDT 1	Longitudinal	Rubber base

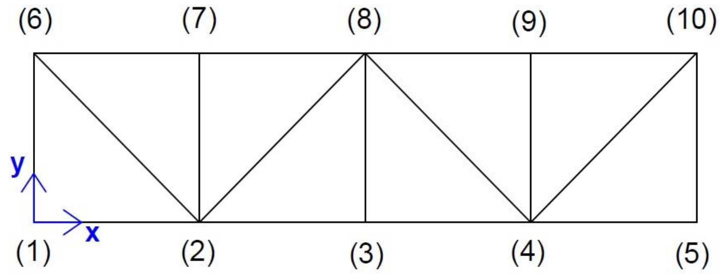


Figure 3: The locations of sensors on top chord of bridge (plan view).

The bridge model was instrumented using wired accelerometer sensors and LVDTs for this series of tests in order to measure structural responses under different excitations. The locations and orientation of the accelerometer sensors are shown in Figure 3. It should be mentioned that *x* and *y* axis represent longitudinal and transverse direction of the bridge model. Table 1 shows characteristics of the wired accelerometers and LVDTs utilised for the tests. This table presents the name of channels, their measurement directions, and their locations on the bridge model. In total, 14 wired accelerometer sensors were attached to the steel joints to measure the structural responses. One wired accelerometer (ACC0) was located on the shake table to measure the ground motions simulated by the shake table. Ten wired accelerometers (ACC1-ACC10) were attached to the steel joints, joint 1 to joint 10, at top chord of the bridge to measure the structural responses in transverse direction. Due to the fact that the wired accelerometers can measure the structural responses in one direction (y direction), two more accelerometer sensors (ACC11-ACC12) were attached to joint 6 and joint 1 in longitudinal direction of the bridge.

To monitor the nonlinearity degree of rubber-based supports, two accelerometer sensors (ACC13-ACC14) and two LVDTs (LVDT0-LVDT1) were installed on one of the supports in both transverse and longitudinal directions as shown in Figure 2(b). The measurements from these sensors have been used to simulate the acceleration-displacement relationships of the rubber-based support under various excitation sources. As mentioned to simulate different degrees of nonlinearity in the structure, the bridge model was subjected to several ground motions with various amplitudes. The characteristics of the excitation sources used during the dynamic tests are given in Table 2, including their name, type, and amplitude.

Table 2: Characteristics of the excitation sources used for dynamic testing.

Name	Type	Amplitude (mm)
Test 1	Sine Sweep Excitation	0.05, 0.1, 0.3, 0.7, 1, 2
Test 2	El Centro Earthquake	0.9, 13, 17, 35, 43, 52, 63, 70, 79, 83, 87, 98
Test 3	Chi-Chi Earthquake	6.5, 13, 26, 54, 100, 130, 190, 210
Test 4	Tabas Earthquake	4, 8, 16, 32, 64, 80, 120, 145, 160, 18, 190

3.2 Data analysis and results

In this part, the analysis results obtained using the vibration-based nonlinearity identification technique are presented using the vibration data measured from the bridge model. First, in order to show the nonlinear behaviour of the rubber-based supports during high amplitude of excitation, the acceleration-displacement graphs obtained from the instrumented rubber-based support are presented. To do so, the bridge model was excited using different amplitudes of sine sweep signal and El Centro earthquake.

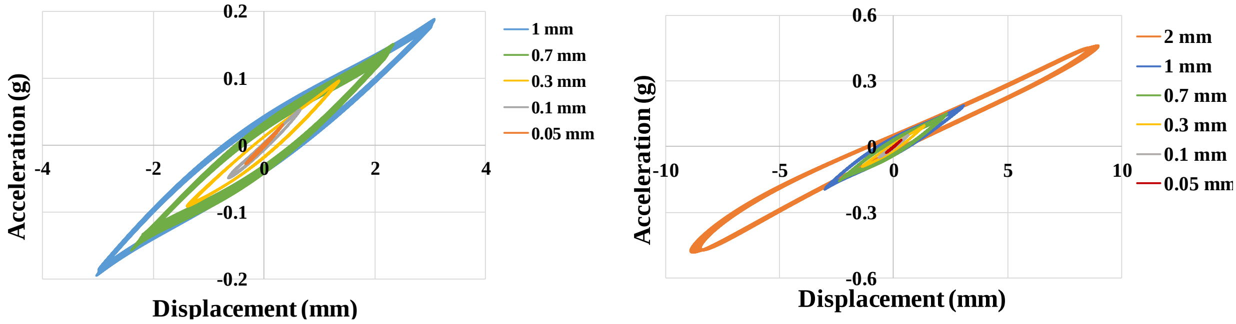


Figure 4: Acceleration-displacement graphs obtained from rubber-based support during Test 1.

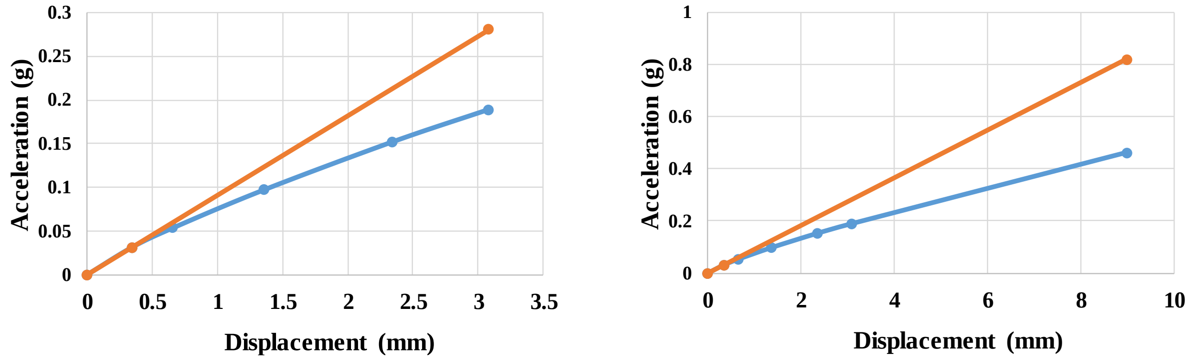
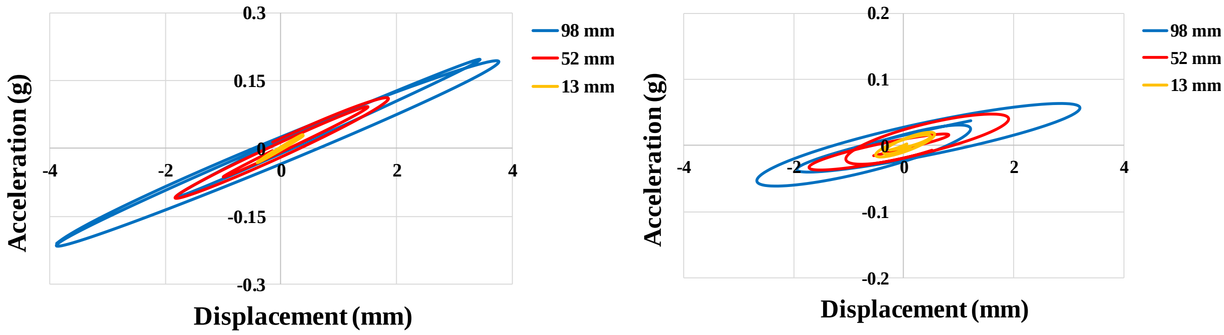


Figure 5: Backbone curves obtained from rubber-based support during Test 1.

Figure 4 shows the force-displacement graphs obtained from the rubber-based support during the sine sweep excitation in transverse direction of the bridge structure. The sine sweep excitation was applied to the structure with different amplitudes started from a minimum of 0.05 mm to a maximum of 2.0 mm and frequency range of 6-6.4 Hz. Figure 5 also presents the corresponding backbone curves of the force-displacement hysteresis loops shown in Figure 4. As presented in these figures, the structure started to behave nonlinearly with the increase in excitation amplitude, as the slope of acceleration-displacement graphs starts to reduce gradually. It can be concluded from the results that the stiffness of the system starts to decrease after an amplitude of 0.1 mm.

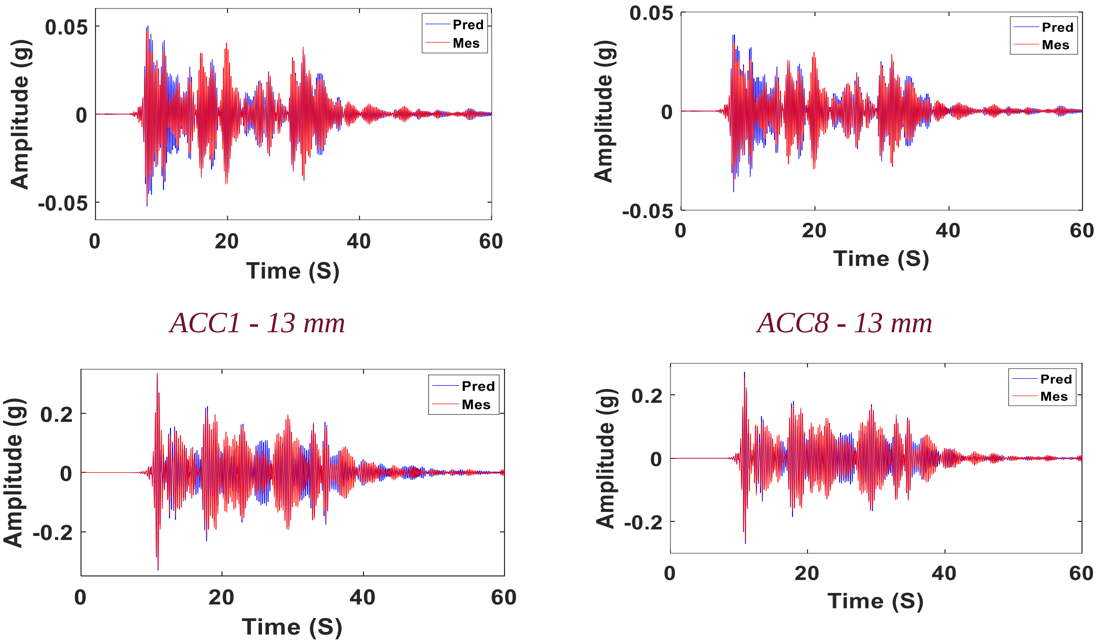


(a) Transverse direction *(b) Longitudinal direction*

Figure 6: Acceleration-displacement graphs obtained from rubber-based support during Test 2.

Figure 6 also presents the acceleration-displacement graphs obtained from the rubber-based support during three different amplitudes of El Centro earthquake in both transverse and longitudinal directions. These results were extracted by considering just a few samples of data with maximum amplitudes. Similar to the sine sweep excitation, the bridge model showed nonlinear behaviour (reduction in stiffness) with the increase in excitation amplitudes. The reduction in slope of acceleration-displacement graphs is more obvious in longitudinal direction of the bridge model. It can be concluded that the rubber-based supports of the bridge model can represent the material nonlinearity type in the structural system. It should be mentioned that this category of nonlinearity is one of the most common nonlinearity types in real-world civil infrastructures, such as bridges. After confirming the presence of the nonlinearity in the structural system using the rubber-based supports, the analysis results obtained using the nonlinearity identification algorithm are presented.

During the dynamic testing, the bridge model was subjected to three well-known ground motions; 1940 El Centro earthquake, 1999 Chi-Chi earthquake, and 1978 Tabas earthquake. A low amplitude of each earthquakes was applied to the bridge structure and assumed to represent its reference linear model. For example, it was assumed that the bridge model has a linear dynamic behaviour under the El Centro earthquake once an amplitude of 0.9 mm was applied to the model. So, this model was considered as a reference linear model to investigate the behaviour of the structure under this ground motion. Different ARMAX models were constructed using the structural responses for different amplitudes. The ARMAX model constructed using the reference linear model was considered Linear model and the other ARMAX models from higher amplitudes of excitation were considered as Unknown models. It should be mentioned that the excitation input recorded using ACC0 located on the shake table was used as the input of time series models.

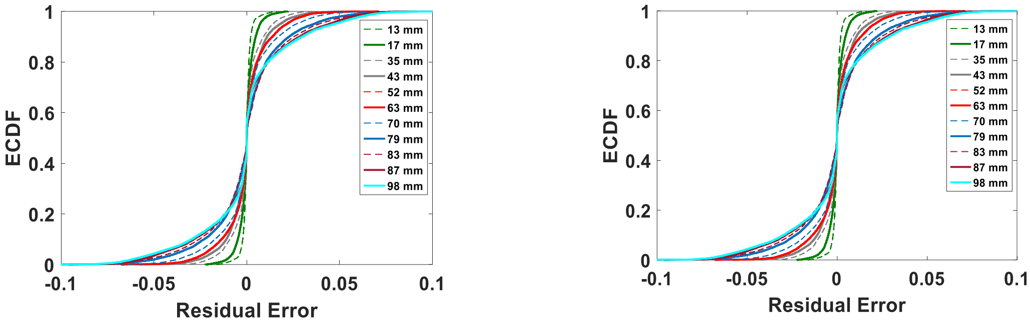


ACC1 - 13 mm *ACC8 - 13 mm*

ACC1 - 98 mm *ACC8 - 98 mm*

Figure 7: Measured and predicted responses recorded during Test 2 in transverse direction.

Figure 7 presents the structural responses measured from the bridge model using ACC1 and ACC8 during the amplitudes of 13 mm and 98 mm of El Centro earthquake. These figures also show the structural responses predicted from the corresponding ARMAX models. A fit ratio of 88.3% and 89.7% was obtained for channels 1 and 8 during the low amplitude of excitation (13 mm). Also, a fit ratio of 78.7% and 82.6% was obtained for channels 1 and 8 during the high amplitude of excitation (98 mm). It can be observed that the ARMAX modelling was able to predict the structural responses with satisfactory prediction and the numerical models are reliable to be used for nonlinearity identification process. The orders of polynomials n_a, n_b, n_c, and n_k for the vibration data sets were set to 4, 4, 1 and 1, respectively.

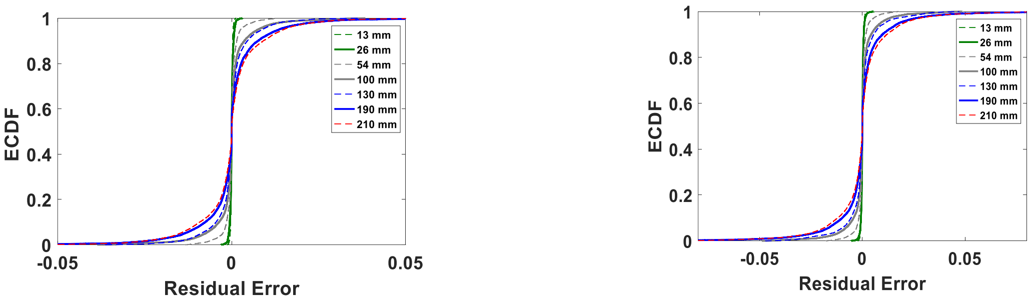


ACC1 - 98 mm *ACC8 - 98 mm*

Figure 8: ECDFs due to various levels of El Centro Earthquake in transverse direction.

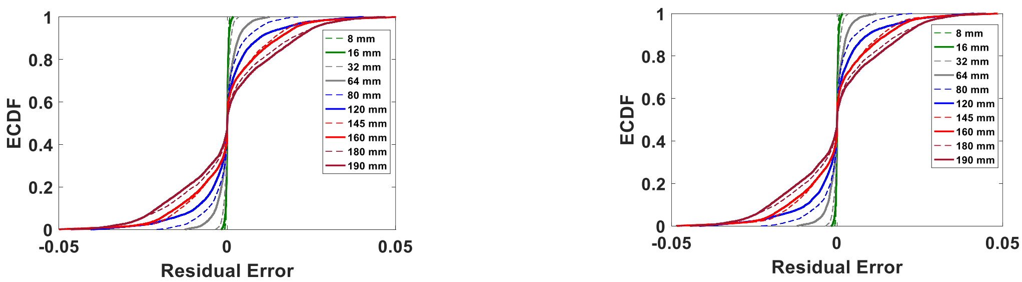
In the next step, the residual errors were calculated for the reference model and the unknown models. The ECDFs of residual errors obtained from different amplitudes of El Centro earthquake using ACC1 and ACC8 are illustrated in Figure 8. The empirical cumulative distribution function of each time series residuals graphically showed that the deviation of residual errors from the reference line increases as the amplitude of excitation increases. This shows the fact that the at higher levels of excitation, the residual errors extracted from linear ARMAX models do not follow a normal distribution.

For Chi-Chi earthquake and Tabas earthquake, it was considered that the bridge model has a linear behaviour under an amplitude of 6.5 mm and 0.4 mm, respectively. Therefore, these models were utilised as the reference linear models to analyse the behaviour of the bridge model under the earthquakes. The structural model was then excited by various amplitudes of these ground motions from a minimum of 13 mm to a maximum of 210 mm for Chi-Chi earthquake and from a minimum of 4 mm to a maximum of 190 mm for Tabas earthquake.



(a) ACC4 *(b) ACC6*

Figure 9: ECDFs due to various levels of Chi-Chi Earthquake in transverse direction.



(a) ACC2 *(b) ACC7*

Figure 10: ECDFs due to various levels of Tabas Earthquake in transverse direction.

Various ARMAX models were constructed for different amplitude cases. A fit ratio of 95.3% and 96.4% was obtained for channels 4 and 6 during the low amplitude of Chi-Chi earthquake (13 mm). Also, a fit ratio of 86.2% and 88.9% was obtained for channels 4 and 5 during the high amplitude of excitation (210 mm). In addition, during the TABAS earthquake, a fit ratio of 98.8% and 98.9% was obtained for the amplitude of 4 mm and a fit ratio of 81.6% and 82.1% was estimated for the amplitude of 190 mm for channels 2 and 7, respectively. Therefore, considering the good fit ratios obtained for low and high amplitudes of earthquakes, the ARMAX models constructed from the measured vibration data sets can be reliably utilised for nonlinearity identification. After calculating the residual errors, the ECDF values of residuals obtained from different amplitudes of Chi-Chi earthquake using ACC4 and ACC6 are shown in Figure 9(a) and (b), respectively. In addition, Figure 10(a) and (b) presents the ECDFs of residuals obtained from various amplitudes of Tabas earthquake using ACC2 and ACC7, respectively. As obvious, the ECDF plots corresponding to higher amplitudes of both ground motions showed more deviation from the reference line. This confirms the fact that the linear ARMAX model residuals do not follow a linear normal distribution as the amplitudes of excitation exceed from a specific range. This range varies for different ground motions with various frequency content. In future study, a new technique based on the results of this paper will be proposed to specify a unique threshold for each subjected earthquake.

4 CONCLUSION

In this study, a vibration-based nonlinearity identification technique is presented to identify the early changes in a dynamic system prior to any significant structural damages. This technique combines vibration data sets with Autoregressive Moving Average with Exogenous Inputs (ARMAX) modelling and probability theory to recognise nonlinearity in a dynamic system. The concept of this method was extracted from the fact that residual error of a linear ARMAX model follows a normal distribution. The technique can categorise the linear and nonlinear behaviours of a structure, when it is subjected to various levels of excitation source. In order to verify the performance of the method, a series of shaking table tests was conducted on a steel truss bridge model in the laboratory environment. Rubber mounts were attached to the bridge supports to simulate material-type nonlinearity in the bridge model. The bridge model was excited using different amplitudes of ground motions to control the nonlinearity degree of rubber-based supports. The vibration data recorded using high-performance accelerometers from the bridge model under different ground motions was used for data analysis. The results of shake table tests obtained using the vibration-based nonlinearity identification technique proved the validity of the method to analyse the earthquake-induced vibration data. The results showed that the vibration-based nonlinearity identification technique is able to identify nonlinear behaviour of the bridge once it was subjected to different levels of earthquake excitations. This algorithm can be very helpful to assess the structural performance at early stage of damage occurrence after sudden events.

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