

# Modelling of residential house perimeter foundation beams subjected to ground deformations

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## ABSTRACT

The 2010-11 Canterbury earthquake sequence is the most damaging and disruptive seismic event to affect New Zealand, resulting in repair costs totalling approximately NZ\$40 billion. Liquefaction induced ground deformations was a distinctive feature in the earthquakes, especially in residential areas of Christchurch. Prior studies have shown that severe damage to the residential houses was more often related to liquefaction induced ground deformation, as opposed to inertial loads associated with earthquake shaking. The Canterbury Earthquake Recovery Authority (CERA) conducted a reconnaissance programme to divide residential land into zones based on observed land performance in the 2010-11 earthquakes. The programme resulted in the technical category (TC) foundation recommendations given in the Ministry of Business, Innovation and Employment (MBIE) guidelines. This study evaluates the inelastic behaviour of several perimeter foundation beam sections subjected to ground deformations, and considers damage implications for the house above. A nonlinear soil-spring bed supporting a nonlinear fibre-element foundation section is validated and used for simulating the soil-foundation interface behaviour of residential houses. TC foundation sections are tested under design performance objectives. Three damage measures were considered: vertical displacement, bending moment, and shear force. Results indicate TC foundations given in the MBIE guidelines perform as expected, based on these three damage measures. Foundation beam size recommendations are provided with the intent to limit damage in the house above, from a drift perspective, when subjected to liquefaction-induced ground deformations.

## 1 INTRODUCTION

### 1.1 Background

The 2010-11 Canterbury earthquake sequence caused extensive damage to light timber-framed residential houses. However, from a life-safety perspective, residential houses typically performed well. Post the earthquake sequence, the financial losses to residential buildings were approximately NZ\$16 billion of the estimated \$40 billion total building and infrastructure losses (Horspool et al., 2016; Reserve Bank of New Zealand, 2016). A distinctive feature of the earthquake sequence was the severity and spatial extent of

earthquake induced liquefaction in native soils. Typical liquefaction ground damage consists of vertical and differential settlements, lateral movements and flooding. Liquefaction induced ground deformation was estimated to have severely affected 15,000 residential properties/buildings, and approximately 5,000 residential properties were estimated to be abandoned (Cubrinovski et al., 2011). Further note that, residential insurance claims reached approximately 130,000 within 90 days after the September 2010 Darfield earthquake (King et al., 2014). Post the earthquake sequence, the Canterbury Earthquake Recovery Authority (CERA) was formed to lead the ongoing reconnaissance. CERA recognised the need to divide Christchurch into zones due to repeated liquefaction damage observed in subsequent aftershocks. Zones were intended to provide residents with certainty and increase community resilience in future earthquakes (Rogers et al., 2013). Zoning resulted in three technical categories, (TC)s.

## 1.2 TC performance objectives

Index criteria for the TC's are shown below in Table 1.

*Table 1: Index criteria for foundation technical categories, (MBIE, 2012).*

<b>Foundation Technical Category*</b>	<b>Nominal SLS Land Settlement</b>	<b>Nominal ULS Land Settlement</b>	<b>Nominal Lateral Stretch</b>
<b>TC1</b>	0-15 mm	0-25 mm	N/A
<b>TC2</b>	0-50 mm	0-100 mm	<50 mm
<b>TC3</b>	>50 mm	>100 mm	>50 mm

\*Where confirmed via site investigation.

The Ministry of Business, Innovation and Employment (MBIE) guidelines on TCs appears to be a result of the aforementioned post-earthquake sequence reconnaissance. The intention of TCs is to increase the resilience of residential dwellings in future earthquakes (MBIE, 2012). MBIE guidelines recommend typical solutions for shallow foundations, which depend on the TC of the site determined through geotechnical investigations. MBIE states that all foundations should be capable of supporting the house above, if loss of foundation support spans 4 m internally or cantilevers 2 m at a foundation edge (MBIE, 2012). Loss of foundation support is associated with loss of bearing capacity due to liquefaction induced ground deformations. Furthermore, for the case of no support over 4 m, MBIE (2012) state that house foundation designs should not hog or sag more than 5 mm. These requirements ensure foundation systems have sufficient stiffness to permit releveling, via jacking at perimeter points, to enable quick repairs in future earthquakes.

## 1.3 Previous Studies

Buchanan et al. (2011) studied the performance of residential houses during the February 22<sup>nd</sup> 2011 Christchurch earthquake. The majority of houses in Christchurch are 1 or 2-storey light timber framed buildings, which performed well from a life-safety perspective. However, thousands of houses had some degree of structural or non-structural damage. Timber framed houses subjected to lateral ground movement and settlement due to liquefaction were often found to be intact, but damage ranged from minor to severe. Severe damage was not always a result of structural collapse, but rather due to a lack of functionality of doors, windows and damage to non-structural elements. For future research, this study recommended to review the

current building code requirements for reinforcing in concrete foundations, especially for sites prone to liquefaction induced ground deformations.

Leeves et al. (2012) conducted a study on rebuilding for resilience in residential housing. Conclusions of this study suggest severity of the damage to residential buildings was related to liquefaction induced ground deformations. Three key objectives in the design philosophy for remediating residential houses arose from this study: 1) negligible damage from liquefaction under serviceability limit state (SLS), 2) maintain life-safety under ultimate limit state (ULS) and, 3) enhanced performance under ULS events such that temporary occupancy can be safely maintained, and houses can be repaired quickly.

Previous work by Henderson (2013), consisted of inspecting approximately 670 residential houses with a focus on local deformations. Results indicate that damage to foundations was closely related to the ground damage the foundation was subject to. Concrete perimeter beam foundations were found to perform poorly from a local deformation perspective, likely due to their flexible nature and lack of robustness in the connection system between foundation and super-structure. Additionally, 2-Dimensional (2D) elastic modelling of the concrete perimeter beam foundation sections on a bed of linear-elastic soil springs subject to a particular uniformly distributed load (UDL) was conducted. This modelling assessed the performance of existing house foundation beams under a range of simplified loading conditions. The 2D modelling was used to compare elastic bending moment demands with corresponding section yield capacities. Elastic modelling means displacements could not be investigated, however, this was highlighted as an area for future research.

## 1.4 Objectives and Scope

The objectives of this study are to:

1. Investigate the inelastic behaviour of perimeter foundation beam sections for residential buildings subject to ground deformations.
2. Evaluate the TC foundation solutions as proposed by MBIE (2012), for a range of house typologies, subject to complete loss of support (LOS) for 4 m of internal span (MBIE, 2012). This requirement ensures foundations have sufficient stiffness to enable releveling via jacking in a future earthquake. Additionally, one foundation beam section from Henderson (2013) is modelled to show a range of damage results which may have occurred in the Canterbury earthquake sequence.
3. Assess the damage implications, attributed to ground deformations, for the house above. Damage in the house above is then used to determine how the TC foundations perform against the design philosophy proposed by Leeves et al. (2012), specifically objectives 2 and 3. Lateral drift damage-states for timber-framed partition walls and brick veneer cladding from literature are used to test the proposed design philosophy.

TC2 and TC3 options that require specific engineering design are considered outside of the scope of this study. Lateral stretch ground deformations were also outside the scope of this study.

## 2 MODELLING METHODOLOGY

Three different foundation perimeter beam sections were modelled in this study, shown in Figure 1 below. TC2 Option 3 and 4 from MBIE (2012), and the 1930-59 foundation beam section from Henderson (2013). All dimensions shown in Figure 1 are in mm.

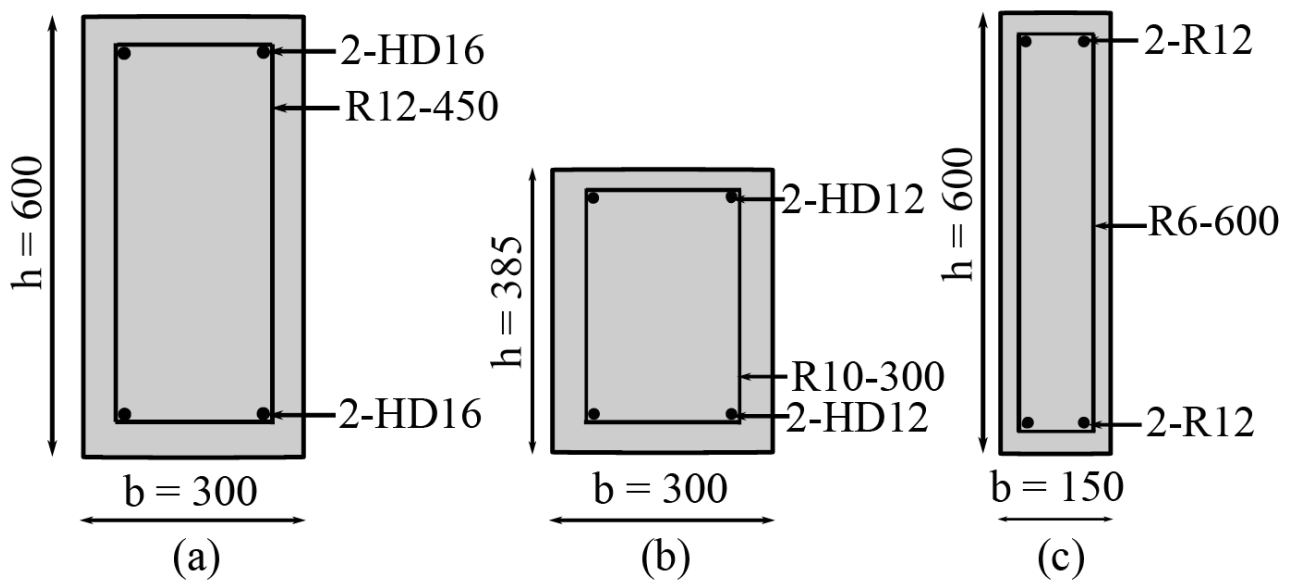


Figure 1: Foundation perimeter beam sections: TC2 Option 3 (MBIE, 2012) (a), TC2 Option 4 (MBIE, 2012) (b), and 1930-59 (Henderson, 2013) (c).

A preliminary sensitivity study was conducted to determine the optimal spring (node) spacing and foundation length. Analyses were conducted with a constant UDL and the spring spacing ( $x$ ) was decreased in increments the same as the methodology of Henderson (2013). These increments were 0.5 m – 0.25 m, then 0.25 m to 0.1 m. Once the difference between iterations was less than 10% for the displacement, bending moment and shear force demands the corresponding value was used in all analyses. Similarly, foundation length ( $L$ ) was increased, in 5 m increments starting at 10 m, until demand results were less than 10% of the prior iteration. Results of the sensitivity study showed the optimal node spacing and foundation length was 0.1 m and 15 m respectively. These values were used for all analyses in this study.

To evaluate the vertical displacement, bending moment and shear force demands on residential house foundation beams, a series of static numerical analyses were performed in OpenSees using O3seespy – an extension library of OpenSeesPy (Zhu et al., 2018). A nonlinear soil-spring bed, with a tributary spacing of 0.1 m was used to model liquefaction induced ground deformations. When subject to a particular UDL, the soil-springs uniformly deform. The fixed end support of the soil-springs within the 4 m length of LOS were then uniformly displaced downwards until they all fully detached from the above foundation fibre-element section (since the springs could not sustain tension). The loading path was applied in this order to resemble the expected loading of house foundations. The nonlinear fibre-element foundation beam consisted of many *dispBeamColumn* elements with 5 integration points per element. The foundation beam elements were inserted between soil-spring nodes to simulate the soil-foundation interface behaviour of residential houses. The *Concrete04* material model was assigned to the confined and unconfined concrete elements. The *Steel01* material model was assigned to the longitudinal reinforcement elements, where response was elastic until yield and had a linear post-yield stiffness, which in this case for all beam sections was 0.01. The fibre-element foundation beam section was validated using moment-curvature analysis. Results from the structural analysis software CUMBIA (Montejo et al., 2007) were used for validation. The OpenSees fibre-element section predicts yield and ultimate moment capacities within 10% of CUMBIA. Table 2 below shows the material properties used in the modelling and validation. The concrete modulus of elasticity and tensile strength was calculated the same as in CUMBIA (Montejo et al., 2007). Figure 2 below compares the moment-curvature results of CUMBIA and the OpenSees fibre-element, for all sections. Note that, shear failure was not explicitly modelled and was only evaluated in post-processing. The design shear capacity from CUMBIA (Montejo et al., 2007) are then compared with associated shear demands obtained in post-processing.

Table 2: Input material properties for modelling and validation.

Beam Section	$f_c$ [MPa]	$f_y$ [MPa]	$E_s$ [GPa]	$h$ [m]	$b$ [m]	$\epsilon_{ult, unconf}$	$\epsilon_{ult, conf}$	$\epsilon_{crush}$	$c$ [mm]
TC2 – Option 3	30	500	200	0.6	0.3	0.0064	0.015	0.004	50
TC2 – Option 4	30	500	200	0.385	0.3	0.0064	0.015	0.004	50
1930 - 59	17.2	300	200	0.6	0.15	0.0064	0.015	0.004	44

Where,  $f_c$  = concrete compressive strength,  $f_y$  = longitudinal reinforcement yield strength,  $E_s$  = steel modulus of elasticity,  $h$  = beam section height,  $b$  = beam section width,  $\epsilon_{ult, unconf}$  = ultimate strain for unconfined concrete,  $\epsilon_{ult, conf}$  = ultimate strain for confined concrete,  $\epsilon_{crush}$  = concrete strain at crushing for confined and unconfined concrete,  $c$  = concrete cover.

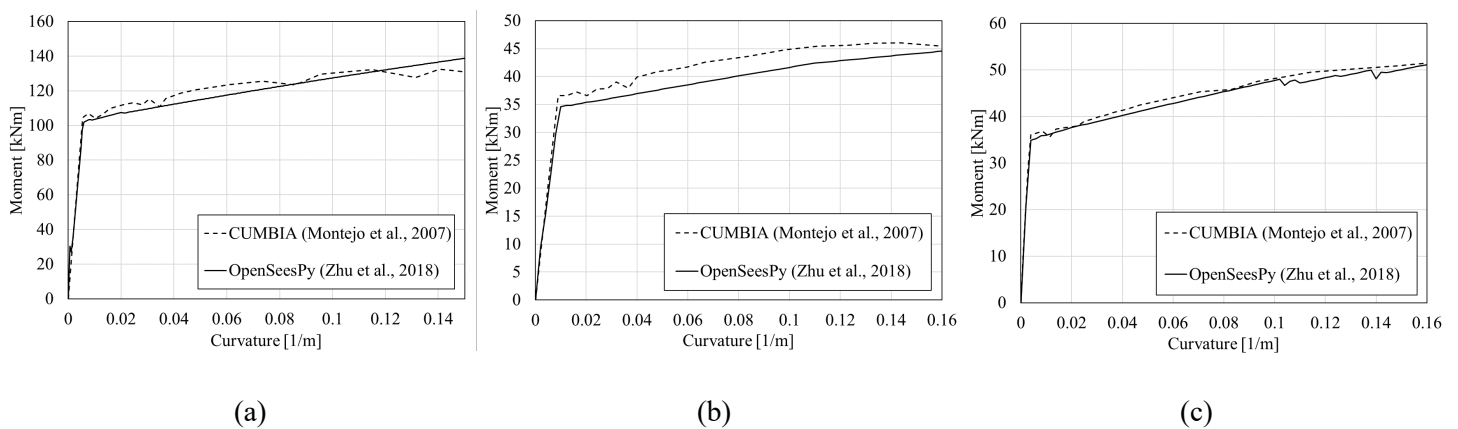


Figure 2: Moment-curvature analysis from CUMBIA and OpenSees for TC2 – Option 3 (a), TC2 – Option 4 (b), and 1930-59 (c).

The soil stiffness and strength were assumed to be uniform for simplicity, however, Loukidis et al. (2017) demonstrated that improved accuracy can be achieved for linear analyses of mat foundation when considering an increase in stiffness at the edge of footings. The coefficient of soil subgrade modulus ( $k_{sub}$ ) was taken as  $k = 25,000 \text{ kN/m}^3$ . This is the same as the modelling methodology of Henderson (2013). Multiplying the soil subgrade modulus by the foundation section width ( $b$ ), and the tributary length, results in the spring stiffness value ( $k$ ). The ultimate soil-spring strength is taken as  $2,500 \text{ kN/m}^2$ , which corresponds to a yield displacement of 0.1 m.

The residential building was represented by a UDL applied as point loads at each node by multiplying by the UDL by the tributary length for the corresponding spring. Different UDL's were considered that represented different residential house weights ( $w$ ). UDL's were considered for 1 and 2-storey houses. House weights for 1-storey buildings were calculated in accordance with NZS3604 (Standards New Zealand, 2011). From Henderson (2013), house weights were calculated by finding the total superstructure weight according to NZS3604 (Standards New Zealand, 2011), and dividing by the perimeter of the house, to give the UDL acting on the concrete perimeter. As per Henderson (2013), 2-storey house weights were determined by accounting for the weight of the first floor, and assuming double the 1-storey weight of ceiling tiles, external wall, external framing and partitions. The different house weights were determined based on the weight of the roof and cladding material used. Four different combinations of roof and wall cladding weights were considered: L/L is light roof and cladding, H/L is heavy roof and light cladding, L/H is light roof and heavy cladding, H/H is

heavy roof and cladding. House weights for 1 and 2-storey residential houses for light/heavy, roof/cladding typologies are shown below in Table 3.

Table 3: Input house weights for different number of stories and roof/cladding types.

Stories – Roof/Cladding	1-L/L	1-H/L	1-L/H	1-H/H	2-L/L	2-H/L	2-L/H	2-H/H
w [kN/m]	4.1	6.2	8.6	10.7	6.9	8.9	16.0	18.1

The 4 m LOS support scenario used in this study corresponds to the upper limit of TC2 nominal ULS land settlement, based on the TC index criteria, given in Table 1. A schematic of the model is shown below in Figure 3.

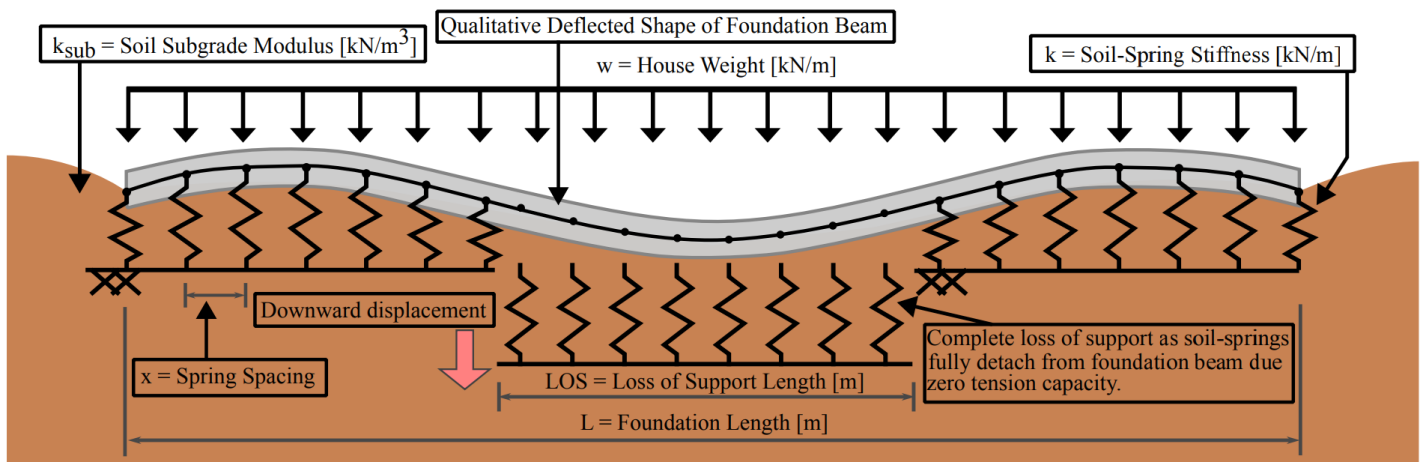


Figure 3: Schematic of the model used in OpenSees, at the final increment of the analysis.

### 3 RESULTS AND DISCUSSION

#### 3.1 TC and existing foundation beam section demands

Figure 4a, b and c below show vertical displacement, bending moment and shear force results for TC2 – Option 3, 4, and 1930-59 perimeter foundation beam sections respectively. These beam sections were subjected to 4 m LOS at the centre of the foundation beam. The respective bending moment and shear capacities are also shown in Figure 3 if within a reasonable range to the demands. The section modulus of elasticity ( $E_c$ ) and modulus of rupture ( $f_t$ ) are calculated the same as in CUMBIA (Montejo et al., 2007) and are shown below in equations 1 and 2 respectively.

$$E_c = 5000\sqrt{f'_c} \quad (1)$$

$$f_t = 0.56\sqrt{f'_c} \quad (2)$$

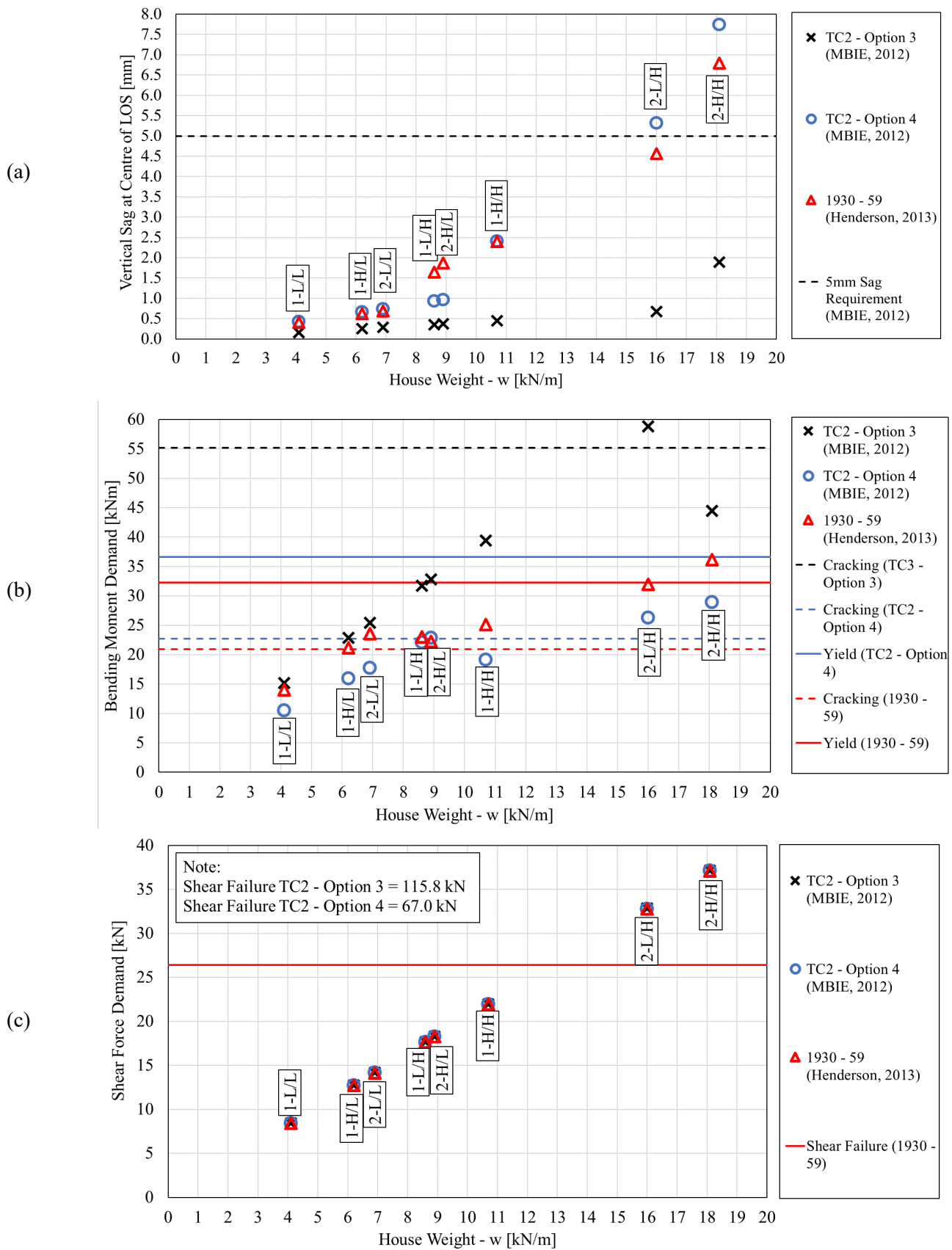


Figure 4: Vertical sag at the centre of LOS (a), bending moment (b) and shear force (c) demand results for TC2 – Option 3, TC2 - Option 4, and 1930-59 perimeter foundation beam sections.

As seen in Figure 4b, TC2 - Option 3 is likely to have section cracking for a 2-L/H (16 kN/m) and 2-H/H (18.1 kN/m) house. Also, no section damage is expected for houses lighter than 2-L/H (16 kN/m). This is generally consistent with MBIE (2012) stating that: reinforcing details are not sufficient for 2-storey houses with a heavy cladding and roof, but can be used for a 2-storey house with a light roof. However, if used for a 2-L/H (16 kN/m) section cracking is likely. Further note, TC2 – Option 3 meets the MBIE 5mm sag requirement for all house weights (Figure 4a).

As seen in Figure 4b, TC2 – Option 4 is likely to have section cracking for houses heavier than 1-L/H (8.6 kN/m). Figure 3a shows that TC2 – Option 4 does not meet the MBIE 5mm sag requirement for 2-L/H (16 kN/m) and 2-H/H (18.1 kN/m). All other house typologies meet the vertical sag requirement. MBIE (2012) state that TC2 – Option 4 reinforcing details are not sufficient for 2-storey houses with heavy cladding and either a heavy or light roof. This is consistent with the results in Figures 4a. However, Figure 4b shows that section cracking can occur in houses as light as 2-H/L (8.9 kN/m). Thus, TC2 – Option 4 should be used with caution.

Figure 4b shows the 1930-59 section cracks, if loaded with a house heavier than 1-H/L (6.2 kN/m). Yield strength is exceeded if loaded with a 2-H/H (18.1 kN/m) house. Although the 1930-59 beam is not designed to comply with the MBIE (2012) 5mm vertical sag requirement, all houses except the 2-H/H (18.1 kN/m) comply with this requirement (Figure 4a).

Figure 4c shows for most house weights and foundation beams, the shear demand is less than the capacity. However, shear failure maybe likely for a 1930-59 beam loaded with a 2-L/H (16 kN/m) or 2-H/H (18.1 kN/m) house.

The peculiar drop in moment demand near the cracking moment observed for all three sections is explained in Figure 5. The displacement and moment demand results for the Option 3 section for the 2-L/H (UDL=16 kN/m) and 2-H/H (18.1 kN/m) are shown, as well as an additional analysis of 14 kN/m. It can be seen that the increase in UDL causes in an increase in the positive moment (tension along top of the beam), while due to a reduction section stiffness in the LOS zone for the 2-H/H (18.1 kN/m) case, the moment associated with the curvature in the LOS zone is less. In other words, there is greater relative cantilever stiffness from the supported beam due to the centre of the LOS softening.

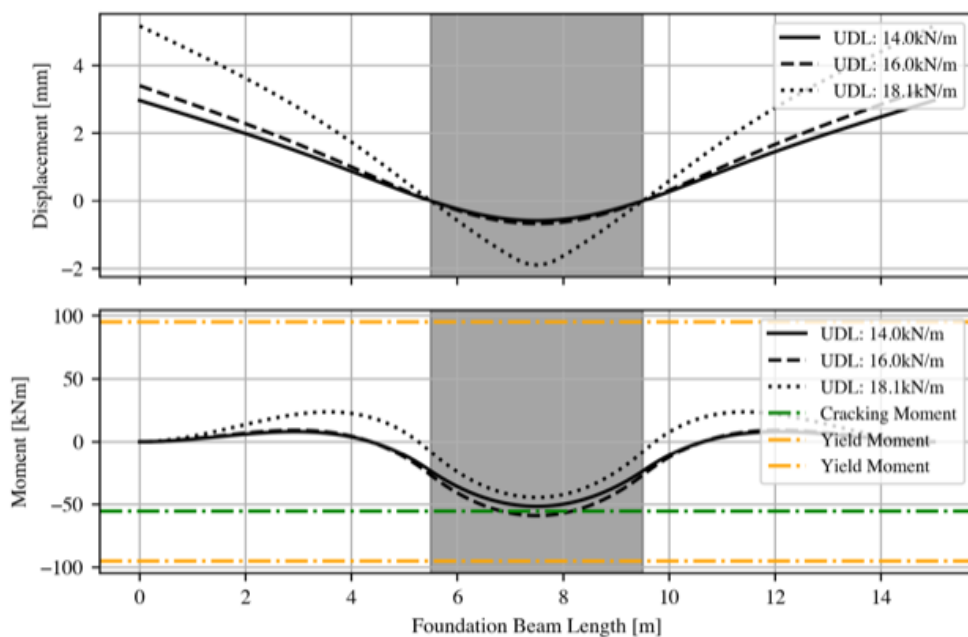


Figure 5: Demonstration of cantilever action for relatively large house weights.



### 3.2 Drift demand on the house above associated with ground deformations

Figure 6 below shows the drift demand corresponding to the simulations above, and corresponding damage implications to the house when subjected to 4 m of LOS at the foundation centre. Drift demand is calculated as an equivalent chord rotation by using the maximum vertical displacement at the centre of the LOS and dividing by half of the LOS length. To assess damage to the gypsum plasterboard walls, drift damage-states from Bhatta et al. (2021) are used. It should be noted, Mulligan et al. (2020) show that minor damage to plasterboard walls can occur at median drifts as low as 0.29%. However, Mulligan et al. (2020) considered the behaviour of a novel partly-sliding plasterboard partition system, hence the 0.29% drift limit is omitted from Figure 6. To assess damage to brick veneer claddings the probable deformation capacity is calculated in accordance with the NZSEE guidelines section C8: Unreinforced masonry buildings (2017). Further note that, Figure 6 only shows drift demands associated with ground deformations.

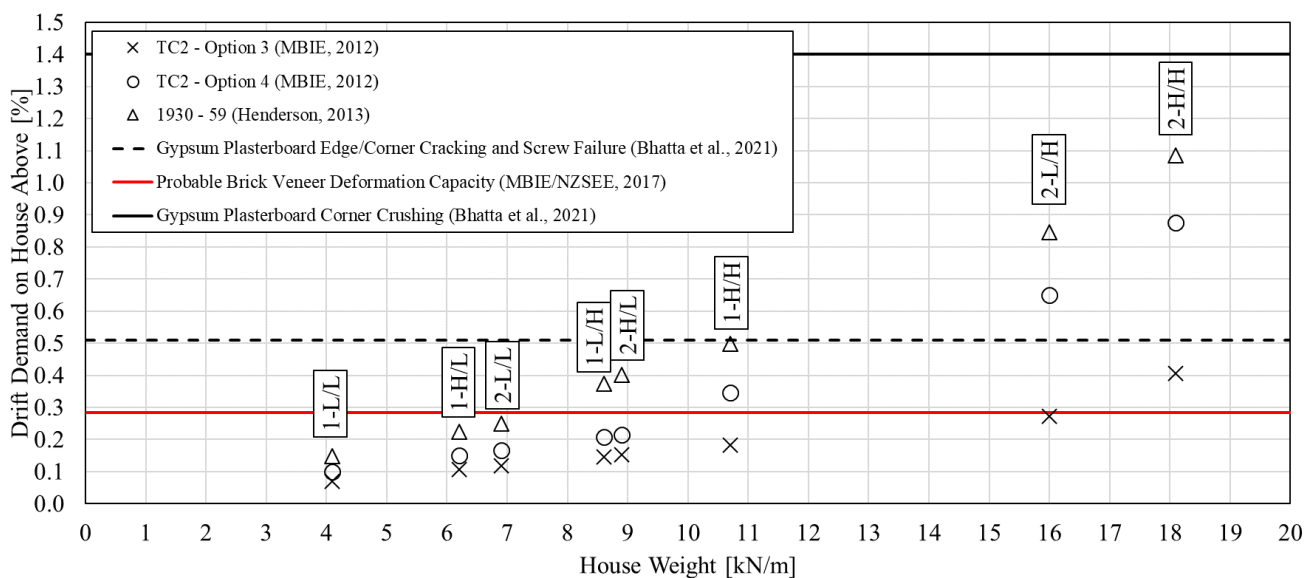


Figure 6: Drift demand associated with ground deformations and damage implications to the house above when subjected to 4m loss of support at the foundation centre.

If TC2 – Option 3 is loaded with a house weight greater than a 2-L/H (16 kN/m), damage to the brick veneers is likely. No damage is expected for the gypsum plasterboard partitions for all house weights. Given MBIE (2012) state that TC2 – Option 3 is insufficient for a 2-H/H (18.1 kN/m), this implies that design objectives two and three proposed by Leeves et al. (2012) are met. As life-safety is maintained at ULS and minor damage indicates the house would be easily repairable.

If TC2 – Option 4 is loaded with a house weight greater than a 2-H/L (8.9 kN/m), damage to the brick veneers is likely. Houses heavier than a 1-H/H (10.7 kN/m) would then also likely have cracking damage to the gypsum plasterboard walls. MBIE (2012) state that TC2 – Option 4 reinforcing details are insufficient for 2-L/H (16 kN/m) and 2-H/H (18.1 kN/m). However, Figure 6 shows that damage to the house above is likely with house weights less than MBIE (2012) recommends. Hence, to comply with design philosophy objectives two and three of Leeves et al. (2012), care must be taken to only use TC2 – Option 4 for relatively lightweight houses.

For houses heavier than a 2-L/L (6.9 kN/m) loaded on the 1930-59 beam, damage to the brick veneer is likely. Houses heavier than a 1-H/H (10.7 kN/m) would likely have cracking damage in the gypsum plasterboard partitions. However, houses on a 1930-59 beam still perform well in terms of life-safety, and damage in the

house above can range from minor to severe. These results are in-line with the conclusions from Buchanan et al. (2011).

Furthermore, the overall system stiffness is expected to increase due to racking behaviour of plasterboard walls. This was not modelled as a part of this study but would enhance the performance of the house-foundation system.

## 4 RECOMMENDATIONS AND CONCLUDING REMARKS

The objectives of this study were to: 1) Investigate the inelastic behaviour of perimeter foundation beam sections for residential houses subjected to liquefaction induced ground deformations, 2) evaluate the TC and existing foundation solutions for a range of house weights, subject to complete LOS for 4 m internal span at the foundation centre, and 3) assess the damage implications for the house above the foundation, and compare TC foundation solutions to the design philosophy proposed by Leeves et al. (2012).

TC2 – Option 3 is recommended as the most suitable perimeter foundation beam for residential houses. TC2 – Option 3 generally performs as expected by the MBIE (2012) guidelines, and complies with the design philosophy proposed by Leeves et al. (2012). TC2 – Option 4 should be used with caution, where the design engineer is confident the house is relatively lightweight. Failing to do so could result in damage to the foundation beam and house above, the potential of requiring repair or a rebuild in a future earthquake also increases. Initial costs associated with using a TC2 – Option 3 section may be slightly higher. However, the consequences, and associated costs for repairs if a TC2 – Option 4 beam is used inappropriately would be much greater in a future earthquake. Performance of the 1930-59 foundation beam was in line with observed behaviour during the Canterbury earthquake sequence.

Future work on this topic should consider: 1) the effect of lateral stretch ground deformation on the foundation, 2) extending the 2D model to the 3D domain to capture the effects of added stiffness from the foundation slab and racking behaviour of plasterboard walls, 3) experimental testing of these foundation beams when subjected to ground deformations, and 4) the performance of TC foundations at other limit states and LOS scenarios considered outside the scope of this study.

## 5 ACKNOWLEDGEMENTS

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