nzsee
NEW ZEALAND SOCIETY FOR EARTHQUAKE ENGINEERING

How to assemble elemental damping?

C.-L. Lee
University of Canterbury, Christchurch, New Zealand.

ABSTRACT

An elemental damping model has recently been proposed as an alternative to the Rayleigh damping model to avoid spurious damping forces during inelastic seismic response. It has shown great potential because it has the same computational efficiency as the Rayleigh model and could better relate damping to elemental deformations. Unfortunately, it has some significant problems. The relationship between elemental and global damping ratios is implicit and peculiar, resulting in difficulty for model parameter calibration. The resultant damping matrix is not classical even if all elemental damping matrices are, causing coupling between global modes with an effect much higher than anticipated. These problems are believed to be caused by the conventional direct assembly method used to assemble elemental damping matrices. This study proposes a more consistent assembly method for assembling elemental damping matrices. The proposed method not only has the advantage of overcoming the above problems but also opens up an opportunity for broader theories that relate damping ratio to internal material state variables, particularly during inelastic response. Examples are used to showcase the performance of the proposed assembly method.

1 INTRODUCTION

In the numerical simulations of seismic response of large-scale structures, the Rayleigh damping model, due to its simplicity and computational efficiency, is often used to incorporate un-modelled energy dissipation. However, since the work by Chrisp (1980) and Carr (1997), it has been proved to result in unacceptable spurious damping forces and moments during inelastic response with an order of magnitude comparable to member forces and moments.

In addressing the issue of spurious damping forces of the Rayleigh model, an elemental damping model has recently been proposed as an alternative (Puthanpurayil et al., 2016; Carr et al., 2017). In this model, elemental damping matrices are formulated based on elemental mass and stiffness matrices. The global damping matrix is obtained by assembling all elemental damping matrices based on the common nodal degrees of freedom. This model has received attention because it shows great potential in better relating damping forces to element deformations and allows analysts to assign different damping ratios from element to element. They are particularly computationally efficient because the resultant global damping matrix is as sparse as the global stiffness matrix.

Unfortunately, the elemental damping model comes with two significant problems: 1) The calibration of elemental damping ratios against desired global damping ratios is difficult, and the relationship between the two damping ratios appears to be peculiar. For a global damping ratio as low as 5%, the required elemental damping ratio could exceed 100% or even be negative. The physical meaning of the elemental damping ratio is lost. 2) The resultant damping matrix is not classical, causing coupling between modes with an effect much higher than anticipated. The interaction between higher and lower mode responses would result in spurious damping moments. It defeats the purpose of using the elemental damping model.

The main reason for the above problems is the conventional assembly method used to assemble elemental damping matrices for a global damping matrix. This study proposes a more consistent method for assembling elemental damping matrices. The proposed method is based on the recently proposed bell-shaped proportional damping model (Lee, 2019a, 2019b, 2020a, 2020b, 2020c, 2021, 2022; Lee and Chang, 2022, 2023). This new method not only has the advantage of overcoming the above problems but also opens up an opportunity for broader theories that relate damping ratio to internal material state variables, particularly during inelastic response. Examples will be used to showcase the performance of the proposed assembly method.

2 ASSEMBLY METHODS

2.1 Conventional direct assembly method

The conventional direct assembly method was proposed by Puthanpurayil et al. (2016) to assemble elemental damping matrices $\boldsymbol{c}$ for the global damping matrix $\boldsymbol{C}$, as defined below

$$\boldsymbol{C} = \bigcup_e \boldsymbol{c} \tag{1}$$

where $\bigcup$ is the assembly operator that combines elemental damping matrices with each element labelled by e and is the standard procedure that assembles elemental stiffness matrices $\boldsymbol{k}$ for the global stiffness matrix $\boldsymbol{K}$ based on the common nodal degrees of freedom shared by the elements. The matrix $\boldsymbol{c}$ is formulated using the elemental mass matrix $\boldsymbol{m}$ and the stiffness matrix $\boldsymbol{k}$. The formulation of $\boldsymbol{c}$ is based on the Rayleigh model, the Caughey model, or the Wilson-Penzien modal damping model. The model parameter values are often calibrated based on user-specified elemental damping ratios.

This assembly method is intuitive but problematic, as discussed in the following:

- Using Equation 1 to obtain $\boldsymbol{C}$ from $\boldsymbol{c}$ would not conserve the elemental damping ratio on the global structural level, such that the global damping ratio is not equal to the elemental damping ratio. As reported previously (Ni et al., 2019), the global damping ratio is often significantly lower than the elemental damping ratio. For matching a global damping ratio of 5%, the required elemental damping ratio could sometimes be way higher than 100%. Such an elemental damping ratio is questionable.
- The relationship between the elemental damping ratio and the global damping ratio is implicit and would change from structure to structure. Some discussions have been presented on how the elemental damping ratios might be calibrated for a desired global damping ratio (Puthanpurayil et al., 2016; Ni et al., 2019; Clemett et al., 2020). However, these calibration methods are not rigorous, particularly for an irregular structure. For a general calibration process, a least-squares curve-fitting method on the global damping ratio could be considered to calibrate elemental damping ratios. This method is, nonetheless, computationally costly since it requires global mode shapes and natural frequencies to be available. They are computationally costly to compute for a sizeable structural model.
- The matrix $\boldsymbol{C}$ obtained using Equation 1 is not classical in general, such that the global vibration modes are coupled. The effects of this coupling could be significant. The interaction between higher and lower modes would result in spurious damping forces and moments, which will be shown in an example later.

2.2 Proposed assembly method

Before the new assembly method is introduced, it should first be noted that the elemental damping matrix $\boldsymbol{c}$ should be formulated based on the recently proposed bell-shaped damping model. The bell-shaped model uses multiple bell-shaped basis functions to generate any desired damping ratio curve in the frequency domain, e.g. a uniform, linear, trilinear or stepped distribution. It is versatile and much more general than the Rayleigh, Caughey, and Wilson-Penzien models. The bell-shaped model has several types of variants for formulating $\boldsymbol{c}$. The following discussions are limited to only the fundamental Type 0 bell-shaped model, but they could be applied to all other types.

The matrix $\boldsymbol{c}$ using the Type 0 model with one basis function is expressed as follows

$$\boldsymbol{c} = \boldsymbol{m_c} - \boldsymbol{m_c}(\boldsymbol{m_c} + \boldsymbol{k_c})^{-1}\boldsymbol{m_c} \tag{2}$$

where

$$\boldsymbol{m_c} = 4\zeta_p\omega_p\boldsymbol{m} \tag{3}$$

$$\boldsymbol{k_c} = \left(4\zeta_p/\omega_p\right)\boldsymbol{k} \tag{4}$$

The parameters ζ_p and ω_p are the coefficients controlling the location and value of the peak of the bell-shaped basis function in order to match a desired global damping ratio curve in the frequency domain. More information about these parameters is given by Lee (2020a).

The expression of $\boldsymbol{c}$ shown in Equation 2 is not suitable to be assembled for $\boldsymbol{C}$. It needs to be first expanded into the following block matrix $\boldsymbol{c_x}$ by introducing additional intermediate degrees of freedom

$$\boldsymbol{c_x} = \begin{bmatrix} \boldsymbol{m_c} & -\boldsymbol{m_c} \\ -\boldsymbol{m_c} & \boldsymbol{m_c} + \boldsymbol{k_c} \end{bmatrix} \tag{5}$$

In this block matrix, the first row corresponds to the original equations of motion, and the second row corresponds to the newly introduced equations that relate a filtering relationship between the original and the intermediate degrees of freedom (Lee, 2020b).

The expression of $\boldsymbol{c_x}$ is then assembled to form the global matrix $\boldsymbol{C_X}$, shown as follows

$$\boldsymbol{C_X} = \begin{bmatrix} \boldsymbol{M_C} & -\boldsymbol{M_C} \\ -\boldsymbol{M_C} & \boldsymbol{M_C} + \boldsymbol{K_C} \end{bmatrix} \tag{6}$$

where

$$\boldsymbol{M_C} = \bigcup_e \boldsymbol{m_c} \tag{7}$$

$$\boldsymbol{K_C} = \bigcup_e \boldsymbol{k_c} \tag{8}$$

Equations 7 and 8 show the key steps for assembling elemental damping. The final global damping matrix $\boldsymbol{C}$ can be obtained by removing all the intermediate degrees of freedom using static condensation, shown as follows

$$\boldsymbol{C} = \boldsymbol{M_C} - \boldsymbol{M_C}(\boldsymbol{M_C} + \boldsymbol{K_C})^{-1}\boldsymbol{M_C} \tag{9}$$

The above equations also apply to cases with multiple basis functions and all types of the bell-shaped model.

A few remarks can be made on this new assembly method:

- The expressions of $\boldsymbol{C}$ and $\boldsymbol{c}$ are similar in form. Therefore, the elemental damping ratio is conserved on the global structural level, such that the global damping ratio is equal to the elemental damping ratio. No complicated calibration of the elemental damping ratio is needed. It is valid for all types of structures.

- The matrix $\boldsymbol{C}$ remains classical so that all global modes are uncoupled if all elements have the same damping ratio. There will be no interaction between modes. No spurious damping forces and moments during inelastic response will result.
- The damping parameter ζ_p can be different from element to element, allowing elemental damping ratios to differ from element to element. It facilitates modelling a structure made of different types of elements, particularly for a structure with its foundation and soil included in the model for soil-structure interaction analysis. Elemental damping ratios can also be assigned to elements without mass inertia and lumped mass elements without stiffness. This flexibility is not allowed in the conventional method.
- The elemental damping ratio no longer needs to be affiliated with elemental mass and stiffness. It can be a function of internal state variables of elements or materials, e.g. plastic strain, damage, or crack. This assembly method facilitates a broader damping theory (Heitz et al., 2019; Chambreuil et al., 2021; Shen et al., 2021, 2022, 2023).
- In computer implementation, the explicit forms of $\boldsymbol{C}$ and $\boldsymbol{c}$ shown in Equations 2 and 9 are unnecessary. Only Equations 6 through 8 are used for high computational efficiency (Lee 2020b).

3 EVALUATION IN RESPONSE HISTORY ANALYSIS

The performance of the proposed assembly method compared to the conventional method is evaluated in a response history analysis of a single-bay ten-storey steel moment frame with a bay width of 6 m and a uniform storey height of 3.5 m. The column and beam sections are W14x193 and W21x68, respectively. The floor weight for horizontal and vertical motions are 50 kN/m and 6.25 kN/m, respectively, and are modelled as beam density in a consistent mass matrix. The first ten shear-mode natural frequencies are 0.442 Hz, 1.40 Hz, 2.58 Hz, 4.00 Hz, 5.73 Hz, 7.75 Hz, 10.0 Hz, 12.3 Hz, 14.3 Hz, and 15.8 Hz. The structure is linear and elastic. The elemental damping model is not yet developed for inelastic structures. A linear elastic analysis is sufficient to show the different performances between the two methods.

The structure is assumed to have 5% damping for all the global modes. In the conventional method, the modal damping coefficients of the beams and columns are computed using the least-squares curve-fitting method to fit the damping ratios of the first ten modes. All the beams have the same damping, while all the columns have the same damping. Nevertheless, the matrix $\boldsymbol{c}$ for the beams can be different from that of the columns. With six modes (three rigid body modes and three deformation modes) in each element, there are twelve free variables (modal damping coefficients) for the curve-fitting. It is often believed that no damping force should resist rigid body motions. Therefore, two options of free variables are considered for the curve-fitting: 1) with rigid body modes (RBM) included, 2) with no RBM. In the proposed method, six Type 0 basis functions are used to generate the uniform 5% damping. The same values of ζ_p and ω_p are assigned to both beams and columns.

Figure 1 shows the damping ratios of all the modes using the two methods. It shows that the conventional method with RBM could match the damping ratios of the first ten modes with acceptable accuracy, but not the option without RBM, which gives very poor matching for the first mode. Both these options also give very large damping ratios for all the higher modes. In contrast, the proposed method successfully gives the correct 5% damping ratio for all the modes with nearly no discrepancy. In fact, since all elements have the same damping ratio, the bell-shaped model performs like the Wilson-Penzien model with all the modes assigned with a 5% damping ratio, as discussed in previous work (Lee, 2020b).

The input motion is the north-south component of the El Centro earthquake. The response history analysis is conducted using the Newmark method with constant acceleration with a time step of 0.02 sec. The resultant roof drift and base shear time histories are shown in Figure 2, with the time window within the period when the motions are large.

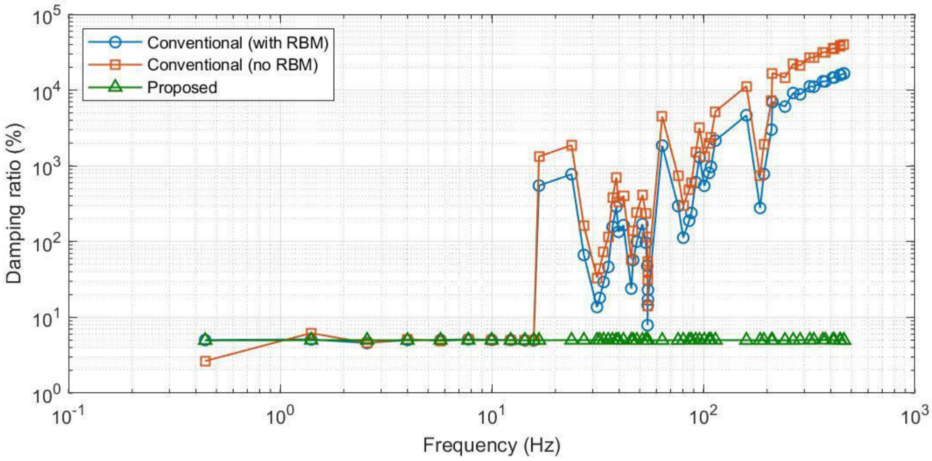

Figure 1: Damping ratios using conventional and proposed methods

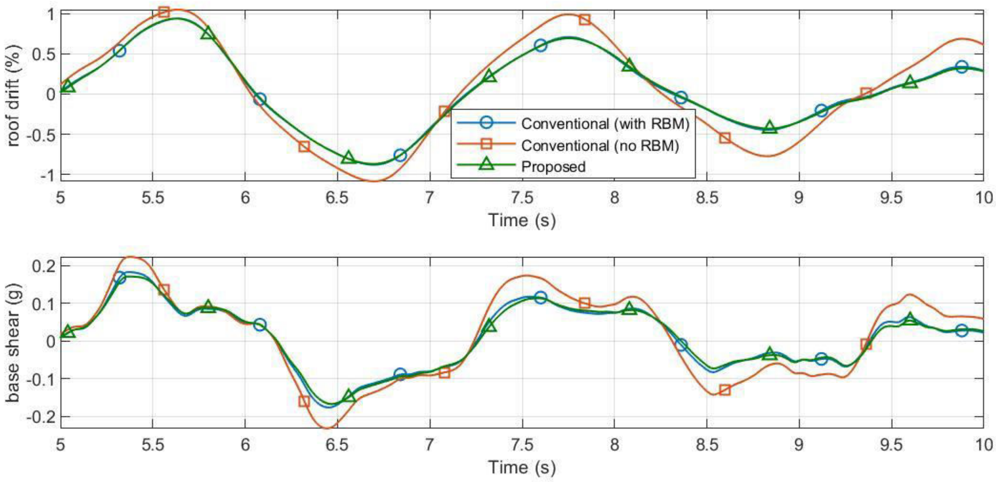

Figure 2: Drift and base shear time histories using conventional and proposed methods

Figure 2 shows that the conventional method with RBM and the proposed method give nearly the same response for the roof drift and the base shear with minor discrepancies because these responses are dominated by the lower modes, for which the conventional method with RBM and the proposed method have successfully produced a 5% damping. In contrast, the conventional method without RBM, due to having a significantly lower damping ratio for the first mode, gives larger roof drift and base shear. These results show that if the conventional method is used, nonzero modal damping coefficients must be assigned to elemental rigid body modes to ensure accurate damping for the lower modes. However, it will result in having unjustifiable damping forces to resist rigid body motions. If the modal damping coefficients for the rigid body modes must be zero, the accuracy of the lower mode damping will be compromised. It is the dilemma of the conventional method.

The envelopes of the inter-storey drifts and the joint damping moments M_{jd} are plotted in Figure 3. The joint damping moments for the left and right columns are identical due to symmetry. The joint damping moments are normalized by the envelopes of the joint member moments M_{j0} (based on constitutive material response) where no damping is present with $\boldsymbol{C} = \mathbf{0}$. A separate undamped response with a zero $\boldsymbol{C}$ has been computed for comparison. As shown in this figure, the conventional method with RBM and the proposed method give a similar envelope for the inter-story drifts. However, the conventional method without RBM gives a larger envelope due to the smaller damping ratio in the first mode. For the joint damping moments

M_{jd}, the proposed method gives an envelope much smaller than the envelope of M_{j0}. This result is good since the damping effect should be minor compared to the member moments. On the contrary, the conventional method gives a much larger envelope, about 4 to 8 times the envelope of M_{j0}, when RBM are excluded from the curve-fitting. When RBM are included, the envelope is about the same as the envelope of M_{j0}, which is still unacceptable. It is because the conventional method gives a very high damping ratio for the higher modes and the coupling effects that result in the interaction between lower and higher modes.

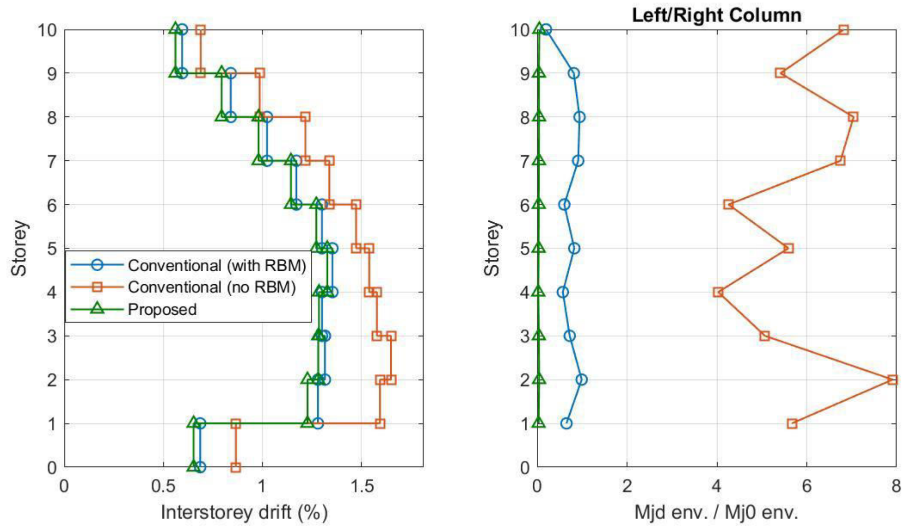

Figure 3: Envelopes of interstorey drifts and joint damping moments using conventional and proposed methods

4 CONCLUSIONS

This paper has summarised the problems of using the conventional direct assembly method for elemental damping models, which include the difficulty of calibration for elemental damping ratios and the unintended large coupling effects between global vibration modes. It has presented a new consistent assembly method based on the recently proposed bell-shaped damping model to address the problems. This new method has not only solved all the problems of the conventional method but also extended the use of the elemental damping model to a wider application using a broader damping theory that relates elemental damping ratios to element or material internal state variables. An example of response history analysis has been presented to show the excellent performance of the proposed method.

5 REFERENCES

Carr AJ (1997). "Damping models for inelastic analyses". *7th Asia-Pacific Vibration Conference*, 9 – 13 November, Kyongju, Korea, pp. 42-48.

Carr AJ, Puthanpurayil AM, Lavan O and Dhakal RP (2017). "Damping models for inelastic time history analysis: a proposed modelling approach". *16th World Conference in Earthquake Engineering*, 9 – 13 January, Santiago, p. 1488.

Chambreuil C, Giry C, Ragueneau F and Léger P (2021). "Locale scale damping model for reinforced concrete elements", *COMPDYN 2021, 8th ECCOMAS Thematic Conference on Computational Methods in Structural Dynamics and Earthquake Engineering*, 28 – 30 June, Athens, pp. 4341-4352.

Chrisp DJ (1980). "*Damping models for inelastic structures*". Master Thesis, Department of Civil and Natural Resources Engineering, University of Canterbury, Christchurch, New Zealand, 42pp.

Clemett NA, Carr AJ and Filiatrault A (2020). “Contributions to the development of elemental viscous damping models”, *17th World Conference in Earthquake Engineering*, 13 – 18 September, Sendai, p. C000729.

Heitz T, Giry C, Richard B and Ragueneau F (2019). "Identification of an equivalent viscous damping function depending on engineering demand parameters", *Engineering Structures*, 188, 637–649. doi:https://doi.org/10.1016/j.engstruct.2019.03.058.

Lee C-L (2019a). “A novel damping model for earthquake induced structural response simulation”. *2019 Pacific Conference on Earthquake Engineering and Annual NZSEE Conference*, 4 – 6 April, Auckland, p. 4C.07.

Lee C-L (2019b). “Efficient proportional damping model for simulating seismic response of large-scale structures”. *COMPDYN 2019*, Vol. 3, pp. 4557–4564. https://doi.org/10.7712/120119.7249.18776

Lee C-L (2020a) “Proportional viscous damping model for matching damping ratios”. *Engineering Structures*, **207**, 110178. https://doi.org/10.1016/j.engstruct.2020.110178

Lee C-L (2020b). “Sparse proportional viscous damping model for structures with large number of degrees of freedom”. *Journal of Sound and Vibration*, **478**, 115312. https://doi.org/10.1016/j.jsv.2020.115312

Lee C-L (2020c). “Proportional viscous damping model for matching frequency-dependent damping ratio”. *17th World Conference in Earthquake Engineering*, 13 – 18 September, Sendai, pp. 2k–0043.

Lee C-L (2021). “Bell-shaped proportional viscous damping models with adjustable frequency bandwidth”. *Computers & Structures*, **244**, 106423. https://doi.org/10.1016/j.compstruc.2020.106423

Lee C-L (2022). “Type 4 bell-shaped proportional damping model and energy dissipation for structures with inelastic and softening response”. *Computers & Structures*, **258**, 106663. https://doi.org/10.1016/j.compstruc.2021.106663

Lee C-L and Chang TL (2022). “Numerical evaluation of bell-shaped proportional damping model for softening structures”. *WCCM-APCOM2022*, 31 July – 5 August, Yokohama, Japan. https://doi.org/10.23967/wccm-apcom.2022.083

Lee C-L and Chang TL (2023). “Implementation and Performance of Bell-Shaped Damping Model”. *Proceedings of the 2022 Eurasian OpenSees Days*, 7 – 8 July, Turin, Italy.

Ni Y, Zhang Z, MacRae GA, Carr AJ and Yeow T (2019). “Development of practical method for incorporation of elemental damping in inelastic dynamic time history analysis”, *2019 Pacific Conference on Earthquake Engineering and Annual NZSEE Conference*, 4 – 6 April, Auckland, p. 4C.11.

Puthanpurayil AM, Lavan O, Carr AJ and Dhakal RP (2016). “Elemental damping formulation: an alternative modelling of inherent damping in nonlinear dynamic analysis”. *Bulletin of Earthquake Engineering*, **14**(8): 2405–2434. https://doi.org/10.1007/s10518-016-9904-9

Shen R, Qian X, Zhou J and Lee C-L (2021). "A time integration method based on the weak form Galerkin method for non-viscous damping systems", *Mechanical Systems and Signal Processing*, **151**: 107361. https://doi.org/10.1016/j.ymssp.2020.107361

Shen R, Qian X, Zhou J, Lee C-L, Carr AJ and Nokes RI (2022), “Study on experimental identification and alternative kernel functions of non-viscous damping in time domain”, *International Journal of Applied Mechanics*, 14(8): 2250062. https://doi.org/10.1142/S1758825122500624

Shen R, Qian X, Zhou J and Lee C-L (2023), “Characteristics of passive vibration control for exponential non-viscous damping system: Vibration isolator and absorber”, *Journal of Vibration and Control*, 10775463221130925. https://doi.org/10.1177/10775463221130925