Building collapse due to P-delta – what is the risk?

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ABSTRACT

Second-order P-delta effects require consideration as part of the seismic design and assessment of buildings, as they can amplify lateral displacement demands and potentially cause collapse through dynamic instability. International codes mitigate the likelihood of P-delta collapse by limiting the value of a P-delta stability coefficient, checked at design intensity levels. However, codes set seismic design provisions in order to limit the annual fatality risk, which is linked to the annual probability of collapse (i.e. the collapse risk) that is in turn related to the design intensity, acknowledging that earthquake shaking can exceed design intensity levels. This paper considers the factors that are likely to affect the collapse risk due to P-delta effects and the implications for fatality risk. By examining the results of non-linear time-history analyses of many single-degree of freedom (SDOF) systems, it is shown that in addition to the so-called P-delta stability coefficient, the hysteretic characteristics of a structure can significantly affect the annual probability of collapse. Collapse fragility functions due to P-delta effects are defined for two different hysteretic models deemed to be representative of well-detailed steel and reinforced concrete buildings. Subsequently, the implications of the results for seismic design are considered. It is argued that future codes should utilise P-delta collapse fragility curves when setting limits to P-delta stability coefficients. This could help ensure that the annual fatality risk associated with dynamic instability is limited to acceptable levels.

1 INTRODUCTION

International seismic design codes (e.g. Eurocode 8, ASCE7-22 and NZS1170.5) tend to permit the analysis of structures for earthquakes using linear, first-order analysis methods. However, because it has long been recognised (e.g. Rosenblueth 1965, Paulay 1978, Bernal 1987, MacRae et al. 1993) that second order P-delta effects can increase displacement demands and risk dynamic instability, codes also prompt designers to compute a P-delta stability coefficient, $\theta_{P\Delta}$, given by Equation 1.

$$\theta_{P\Delta} = \frac{P\Delta_0}{VH} \tag{1}$$

where, as shown in Figure 1, P is the force due to gravity acting on the structure, Δ_0 is the lateral displacement (at the centre of mass) obtained from a first order analysis, H is the height of the structure, and V is the lateral resistance offered by the structure at the displacement Δ_0.

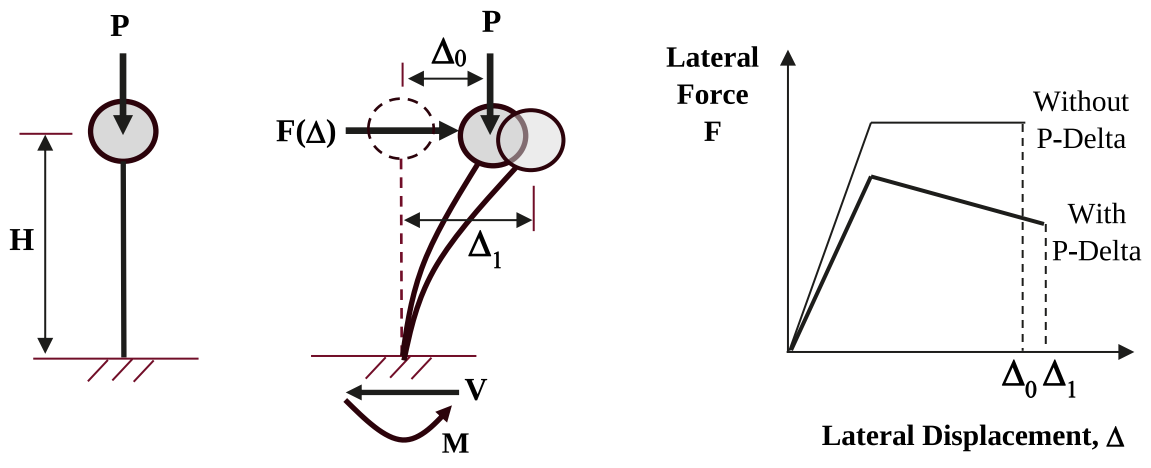

Figure 1: Illustrating the impact of second-order P-delta action on the lateral force-displacement response (right side of figure) of a SDOF system.

The right-side of Figure 1 also illustrates that during an earthquake, P-delta effects tend to increase the peak displacement demand, to an amount shown as Δ_1. The ratio of Δ_1 to Δ_0 is referred to here as a displacement amplification ratio, α_Δ. The general relationship between the displacement amplification factor and the P-delta stability coefficient is shown in Figure 2. For low values of stability coefficient, P-delta effects are negligible. However, for high values of P-delta stability coefficient, dynamic instability can occur, in which the structural system is unable to return to its original undeformed state because of the second-order overturning demand.

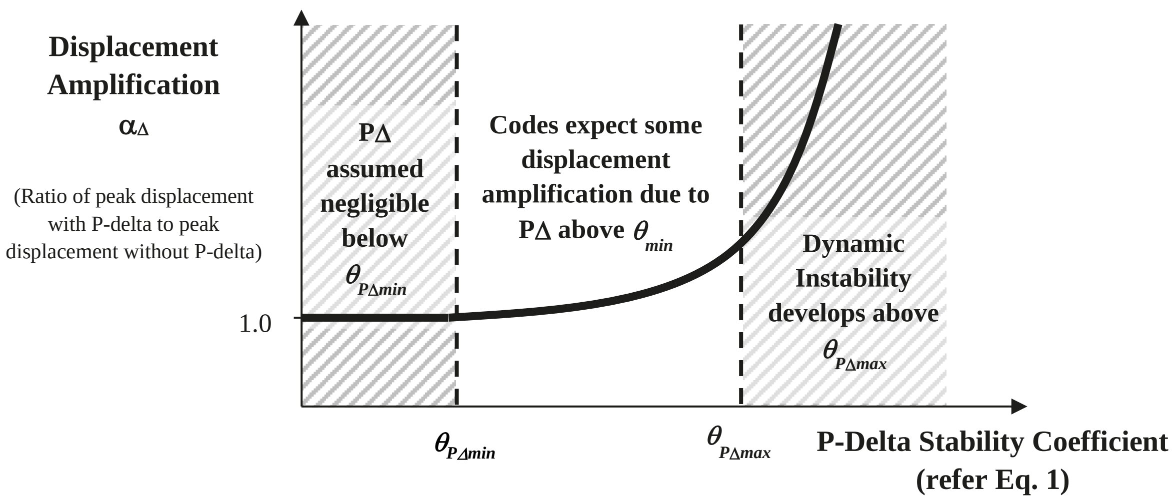

Figure 2: General relationship between displacement amplification and P-delta stability coefficient (After De Francesco and Sullivan, 2023a).

Once a designer has computed the P-delta stability coefficient, seismic design codes then usually prompt mitigation of P-delta effects in the following two ways:

- Amplification of displacement demands (and hence strength and stiffness requirements) for a P-delta stability coefficient that exceeds a minimum value ($\theta_{P\Delta min}$ in Figure 2), typically taken as 0.10.
- Checking that the P-delta stability coefficient obtained at the design intensity does not exceed a maximum value ($\theta_{P\Delta max}$ in Figure 2), typically taken as 0.30.

The latter of these checks, that the stability coefficient does not exceed a maximum value, is implemented in codes to limit the risk of building collapse due to dynamic instability. There is an expectation that for shaking at the design intensity, dynamic instability will be unlikely because of this code check. However, it is also recognised that codes rely on buildings possessing some reserve capacity for shaking intensity levels that significantly exceed the design intensity. As P-delta stability coefficients increase with increasing earthquake shaking intensity (since the denominator in Equation 1 tends to remain constant post-yield), dynamic instability of the structure becomes more likely for higher intensity shaking. With this in mind, the factors affecting the likelihood of collapse due to dynamic instability are examined in this paper and the implications for seismic design are considered.

2 NUMERICAL INVESTIGATION INTO P-DELTA EFFECTS

Results from a recent investigation into P-delta effects by De Francesco and Sullivan (2023) are used to examine the factors affecting the likelihood of collapse due to dynamic instability here. In the work of De Francesco and Sullivan (2023a,b), an ensemble of 1964 ground motions were used to conduct non-linear time-history analyses of SDOF systems with either bilinear, Takeda, Sina or Flag-shaped hysteretic models. In this work, the results for the Bi-linear and Takeda models, illustrated in Figure 3, are examined because these could be representative of well-detailed steel and reinforced concrete structures, respectively. The post-yield stiffness ratio, r, for each model was taken as 0.05. For each hysteretic model, SDOF models were generated with periods of vibration ranging from 0.5s to 4.0s, four different effective heights, and yield strength coefficients (defined as the ratio of the base shear at yield to the building weight) ranging from 0.025 to 0.50. Each model was subject to non-linear time-history analysis using a first- and second-order analysis regime, with a tangent-stiffness proportional damping model and a damping ratio of 5%. A novel implementation of the tangent-stiffness damping model was used to avoid inappropriate damping forces in the post-yield range, as explained in De Francesco and Sullivan (2022).

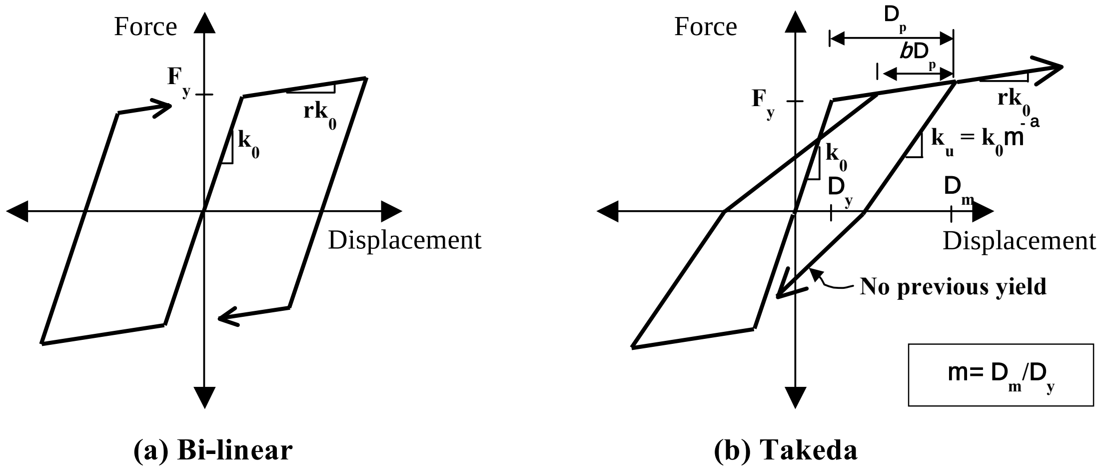

Figure 3: Force-displacement hysteretic models considered herein; (a) Bi-linear and (b) Takeda models. Figure adapted from Stafford et al. (2016).

The analyses run by De Francesco and Sullivan (2023) generated a large dataset of results. Figure 4 illustrates the displacement amplification factors obtained by De Francesco and Sullivan (2023b, 2023c) for the bilinear hysteretic model. The grey diamonds indicate the results obtained for individual ground motions whereas black dots represent the median amplification obtained for different ranges of P-delta stability coefficient (defined according to Equation 1), with a bin size of 0.05 considered. In order to highlight the shape of the data, Figure 4 plots the results to a maximum P-delta stability coefficient of 0.4 and amplification factor of 5.0 even though the full data set includes data out to stability coefficients beyond 1.0 and very high amplification factors (indicative of collapse due to dynamic instability).

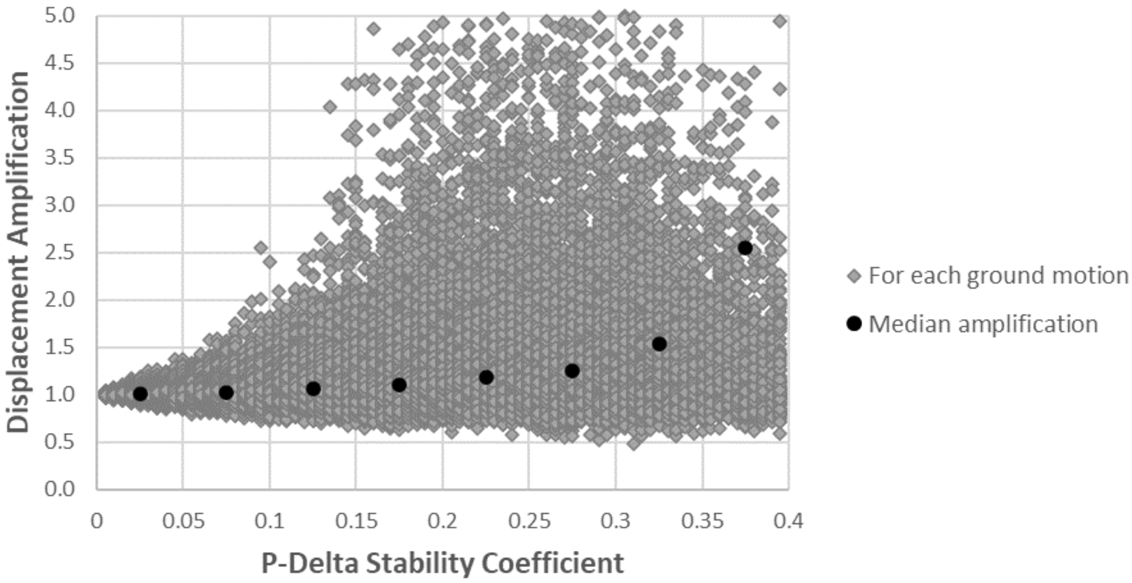

Figure 4: Displacement amplification factors obtained by De Francesco and Sullivan (2023b, 2023c) for SDOF systems with bi-linear hysteretic force-displacement characteristics.

3 P-DELTA COLLAPSE FRAGILITY CURVES

In order to quantify the annual probability of collapse due to P-delta effects, collapse fragility curves are sought. Interestingly, whilst many different collapse fragility curves for buildings can be found in the literature, the authors did not find any collapse fragility curves specifically associated with P-delta induced dynamic instability. Such curves are considered particularly relevant for very well detailed structures that may have been designed with large design ductility values, and low overturning resistances, owing to inherently large ductility capacities.

In this work, collapse due to P-delta induced dynamic instability is assumed to occur when a displacement amplification ratio of five or more was recorded. In other words, collapse due to P-delta was deemed to occur when the NLTH analysis with P-delta effects generated a peak displacement demand five times (or more) greater than the peak displacement demand obtained from NLTH analysis without P-delta effects. Using this criterion, the data generated by De Francesco and Sullivan (2023) is utilised here to generated collapse fragility curves as a function of the P-delta stability coefficient. Data was grouped into stability coefficient bins with intervals of 0.05. The number of collapses recorded and the total number of analysis results for each bin are reported in Table 1. These numbers were then used to compute the probability of collapse due to dynamic instability as a function of the P-delta stability coefficient. Subsequently, the maximum likelihood method was used (in line with the recommendations of Baker, 2015) to establish collapse fragility curves as a function of P-delta stability coefficient, as illustrated in Figure 5. The resulting fragility functions are characterised by median values of P-delta stability coefficient of 0.44 and 0.65 for the Bi-linear and Takeda hysteretic models respectively, together with dispersion values equal to 0.40 and 0.35, again respectively for the Bi-linear and Takeda hysteretic models.

Table 1: Summary of collapse data from NLTH analyses by De Francesco and Sullivan (2023c). Note that data has been binned within each of the stability coefficients shown using a bin size of 0.05.

	Bi-linear Hysteretic Model			Takeda Hysteretic Model		
Stability Coefficient, $\theta_{P\Delta}$	**No. of analyses**	**No. of collapses**	**Prob. of collapse**	**No. of analyses**	**No. of collapses**	**Prob. of collapse**
0.025	75395	0	0.000	70890	0	0.000
0.075	22245	0	0.000	21333	0	0.000
0.125	11899	0	0.000	11303	0	0.000
0.175	7670	32	0.004	7211	3	0.000
0.225	4974	175	0.035	4886	3	0.001
0.275	4410	464	0.105	4276	26	0.006
0.325	3069	788	0.257	2958	53	0.018
0.375	1845	833	0.451	1760	99	0.056
0.425	1582	902	0.570	1474	160	0.109
0.475	1161	739	0.637	1095	197	0.180
0.525	768	571	0.743	699	210	0.300
0.575	840	609	0.725	770	299	0.388
0.625	793	563	0.710	725	322	0.444
0.675	601	475	0.790	588	316	0.537
0.725	504	411	0.815	505	319	0.632
0.775	459	381	0.830	395	266	0.673
0.825	330	302	0.915	360	283	0.786
0.875	234	217	0.927	253	199	0.787
0.925	218	207	0.950	198	157	0.793
0.975	169	159	0.941	135	112	0.830

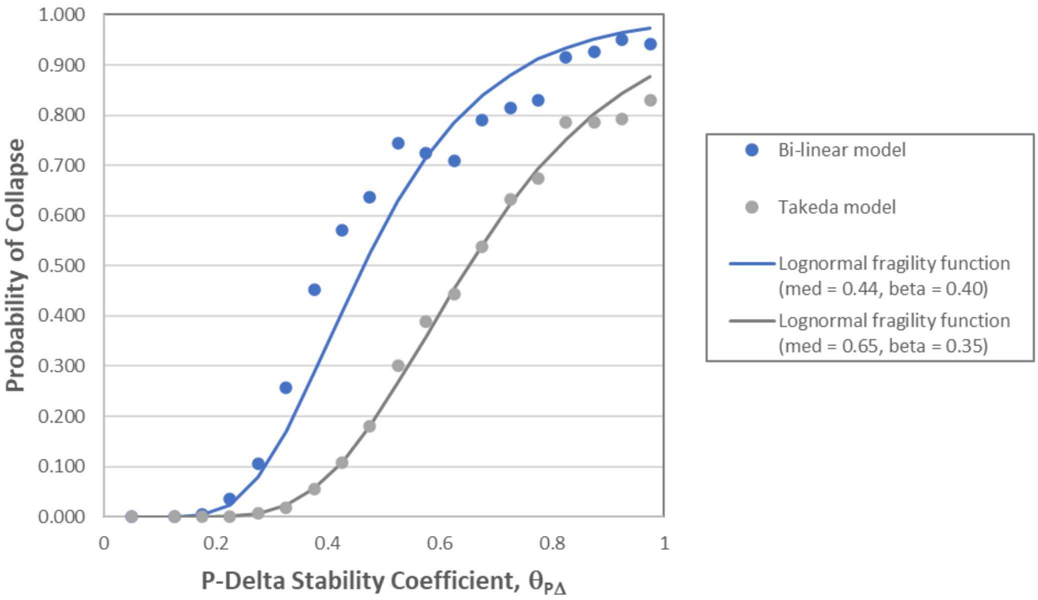

Figure 5: Displacement amplification factors obtained by De Francesco and Sullivan (2023) for SDOF systems with bi-linear (left) and Takeda (right) hysteretic force-displacement characteristics.

4 IMPLICATIONS FOR SEISMIC DESIGN

The fragility functions generated in the previous section could be quite useful for future code writers. The first apparent result is that structures with bi-linear hysteretic characteristics, such as steel structures, are more prone to P-delta instability than structures with Takeda hysteretic characteristics. This agrees with results reported in prior studies and can be anticipated for reasons discussed by Priestley et al. (2007). The second observation one might make is that the imposition of a code maximum limit to the P-delta stability coefficient equal to $\theta_{P\Delta,max}$ = 0.30 (refer Figure 2) may seem reasonable at first, since there appears to be only a 1 in 10 chance that dynamic instability would occur at this level of demand (and even lower chances for the Takeda model). However, recall that code writers assume that structures possess reserve capacity to resist shaking intensity levels well beyond the design seismic intensity. Thus, it is argued that a better means of judging whether the P-delta stability coefficient is suitable is to evaluate what this would imply in terms of annual probability of building collapse.

To compute the annual probability of collapse due to P-delta effects (i.e. dynamic instability), one can combine a fragility function with a hazard curve and integrate over the relevant range of intensity levels. As hazard curves provide the annual probability of exceeding different levels of shaking intensity (and not P-delta stability coefficients), one requires a relationship between shaking intensity and P-delta stability coefficient in order to use the fragility curves in Figure 5. For simplicity, one could assume that the P-delta stability coefficient will increase approximately in proportion to the shaking intensity (similar to the equal-displacement rule). If this is done for a hypothetical building in Wellington, designed to the stability coefficient limit of 0.30 at the ULS design intensity, then integrating the P-delta fragility curves in Figure 5 (scaled so that the stability coefficient equals 0.30 for the ULS design intensity level) with the hazard information from the latest NZ seismic hazard model (sourced from the GNS website with a period T = 0.5s and Vs30 = 400m/s), one obtains an annual probability of collapse due to P-delta of 1.2×10^{-3} for steel buildings and 3.3×10^{-4} for RC buildings. This process is illustrated in Figure 6.

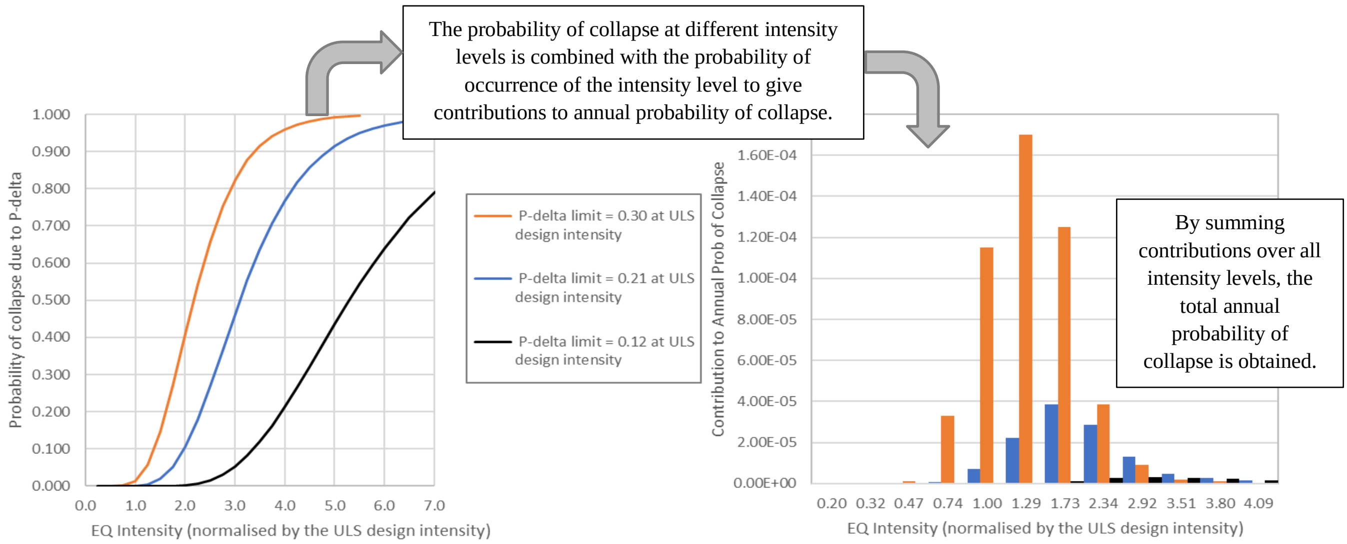

Figure 6: Fragility functions for P-delta collapse of RC structures (Takeda model) as a function of EQ intensity assuming different design limits for the P-delta stability coefficient (left) and contributions to the annual probability of collapse (right) obtained by combining the fragility functions with hazard information.

In reality, it is expected that first-order displacement demands will tend to increase more quickly with increasing intensity than at lower levels of shaking and thus, the annual probability of collapse may be even higher than the values reported above. Given that the commentary to NZS1170.5 suggests that the annual probability of collapse should be less than $1\text{x}10^{-4}$, it is therefore quite evident that a stability coefficient limit of 0.30 may not be limiting the annual probability of collapse adequately. Given this, Figure 6 also includes fragility functions that might be expected if two alternative design limits of $\theta_{P\Delta,max} = 0.21$ and $\theta_{P\Delta,max} = 0.12$ were imposed on the P-delta stability coefficient. The two alternative limits trialled in Figure 6 have been set (for the assumed hazard model) to achieve annual probabilities of collapse of $1\text{x}10^{-4}$ or $1\text{x}10^{-5}$ (for the limits of 0.21 and 0.12 respectively). The lower limit has been trialled because it is argued that acceptable annual collapse rates due to dynamic instability should be set considerably lower than collapse rates due to localised loss of structural (or non-structural) deformation capacity. This is because the likelihood of fatalities conditional on collapse will be much higher in the case of dynamic instability than for the case of localised structural failures. As such, if an annual probability of collapse of $1\text{x}10^{-4}$ is thought to be acceptable for localised structural failures, then the annual probability of collapse due to P-delta (dynamic instability) should be lower, say $1\text{x}10^{-5}$. Table 2 presents the limiting values of stability coefficient that would need to be maintained at the ULS design intensity (assumed to have an annual probability of exceedance of 1 in 500 years) for either $1\text{x}10^{-4}$ or $1\text{x}10^{-5}$ for the bilinear and Takeda hysteretic models. It is seen that design limits for the P-delta stability coefficient would need to be considerably lower than the value of 0.30 used in current codes. Such limits may begin to become critical in seismic design (noting that the stability coefficient is not usually critical in practice currently) and hence, it is recommended that the collapse risk due to P-delta effects should be considered carefully when drafting future seismic design standards.

Table 2: Relating limits for the P-delta coefficient to be checked at the ULS design intensity, with the target annual collapse probability for a hypothetical building in Wellington.

	Bi-linear	**Takeda**
$\theta_{P\Delta max}$ for annual collapse probability = $1\text{x}10^{-4}$	0.13	0.21
$\theta_{P\Delta max}$ for annual collapse probability = $1\text{x}10^{-5}$	0.08	0.12

5 CONCLUSIONS

Using the results of an extensive numerical investigation by De Francesco and Sullivan (2023), this paper has highlighted the factors that are likely to most affect the collapse risk due to P-delta effects and considered the implications for seismic design. The SDOF models used for the NLTH analyses by De Francesco and Sullivan (2023) included Bilinear and Takeda hysteretic characteristics (representative of well detailed steel and reinforced concrete buildings respectively), periods of vibration that varied from 0.5s to 4.0s and a range of strength coefficients. The NLTH analyses were conducted for a large set of recorded ground motions. Results indicate that in addition to the P-delta stability coefficient, the hysteretic characteristics of a structure can significantly affect the annual probability of collapse. Furthermore, by examining the rate of collapse as a function of P-delta stability coefficient, new fragility functions associated with P-delta dynamic instability have been obtained. Subsequently, the implications of the findings for seismic design have been considered. It is argued that design limits for the P-delta stability coefficient should be set with consideration of the collapse risk that would result. It is proposed that this collapse risk can be quantified by combining P-delta collapse fragility curves with hazard information. By trialling this approach for a Wellington hazard curve, it is found that the current stability coefficient limit of 0.30 adopted in most international codes may lead to unacceptably high collapse risk probabilities. By reducing the limit to around 0.2 or 0.1, the annual probability of collapse could be reduced to around $1x10^{-4}$ or $1x10^{-5}$ respectively, with the latter value arguably more appropriate for control of fatality risk.

The process explored in this work for the definition of P-delta collapse fragility functions and design limits for stability functions could be examined further before adoption in practice. One aspect that could be examined is whether the range of SDOF systems examined by De Francesco and Sullivan (2023a,b) adequately represents the building stock in New Zealand, because this might impact the fragility functions obtained. Two specific aspects to include would be: (i) the effective height of the SDOF models (noting that the dataset from De Francesco and Sullivan was generated for effective heights of 1m, 4m, 20m and 40m) and (ii) the contribution of gravity framing and other secondary load paths to the post-yield stiffness ratio (shown as *r* in Figure 3), which was taken as 0.05 in this study. In addition, this work has focussed on the behaviour of SDOF systems because it is assumed that the first mode response dominates the displacement and overturning demands on multi-degree of freedom (MDOF) structures. To this extent, note that seismic design standards often check P-delta stability coefficients on a storey-by-storey basis whereas the limits obtained in this work correspond to the equivalent SDOF system demands. As such, future research should also investigate whether there is a need for storey-based limits or whether equivalent SDOF limits to the stability coefficient would be adequate. Furthermore, some differences in P-delta collapse fragility may result from the examination of MDOF systems compared to SDOF systems, and so this could be examined as part of future research too.

6 ACKNOWLEDGEMENTS

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