Pre-Assessment of Real-time Hybrid Shake Table Test for Evaluation of Seismic Response of MDOF System

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ABSTRACT

A real-time hybrid shake table test (RTHSTT) method is evolved to investigate response of the large structures, which otherwise cannot be tested due to limitations of testing setups. In RTHSTT part of the structure is physically tested (PS) on a shake table while rest of the portion is simultaneously analysed numerically (NS). The key components of the RTHSTT are the controller, which maintains the flow of information between NS and PS, and efficient stable numerical algorithms for real time analysis of NS. The execution of RTHSTT of any structure requires a shake table, physical model of part of the structure, controller, and high computation framework for the analysis of the numerical structure in real-time. Pre-assessment of RTHSTT is a framework useful for validation of the sub-structuring approach and testing of numerical algorithms for real time analysis of NS. In pre-assessment framework working of shake table, PS, NS, and controller are emulated through simulation.

The study focuses on the pre-assessment of substructure shake table tests of 10 story building subjected to earthquake ground motion. In the present paper, equations of motion of PS and NS of 10 story building are formulated in a state-space state. For emulation of RTHSTT, NS, shake table, control system, and PS are numerically formulated in MATLAB Simulink platform. The delays due to shake table dynamics and computational are considered in the simulation. The delay compensation is incorporated to mitigate these delays through PID control. The performance of RTHSTT simulation is assessed with reference to numerical response of 10 story building. This research contributes to development of more robust seismic testing protocols and experimental testing of multi-story structure.

Keywords: *Real-time hybrid shake table test (RTHSTT), Numerical algorithm, control system, Seismic Response*

1 INTRODUCTION

Real-Time Hybrid Shake Table (RTHST) offers a realistic and economical way to study the response of large structures under earthquake loading (Mosalam and Gunay, 2013). The hybrid simulation is also useful to study the response of structures comprising structural elements with different force-deformation characteristics

(Nakashima et. al., 1992). The RTHST test is a close-loop test procedure which decouples the large structural system into physical (PS) and numerical (NS) sub-structures. In RTHST, the PS is tested on or using shake table for the input from NS. The measurements (output) from PS are simultaneously used to solve governing equation of motion of NS (Lee et al., 2007). The sub-structuring method allows testing in laboratories with limited testing facilities (Mosalam and Gunay, 2013).

The critical aspect of RTHST test are to ensure the stability of the numerical integration scheme and the timely transfer of information from NS to PS. The controller manages inputs and outputs signals, compensates for disturbances, and minimizes errors through continuous feedback adjustments (Shao et al., 2010).

In real-time hybrid simulation, delays can arise from communication, computation, and shake table dynamics. The communication delay refers to the time taken to convert signals between analog and digital forms during the synchronization of the physical and numerical substructures. Communication delays can occur in both NS and PS. The computational delay occurs during the evaluation of the next step response by a numerical integration scheme. The delay due to shake table dynamics is finite duration between the signal input to the shake table actuator and execution of motion of the shake table. (Horiuchi et al., 1999, Bonnet et al., 2006). The delay causes phase lag (non-synchronisation) between motions of NS and PS substructures. The delay effect is represented in terms of negative damping. The system response diverges and results in instability (MacRrum and Williams, 2016) when the magnitude of negative damping exceeds the structural damping.

Delay compensation techniques, such as adaptive delay compensation, constant delay compensation, etc., are developed to mitigate the delays and ensure stability of RTHST tests. The delay compensation techniques achieve accuracy through polynomial extrapolation, phase lead compensators, and PID controllers (Chen and Ricles, 2010, Horiuchi and Konno, 2001 Ahmadizadeh et al., 2007 and Zhang et al., 2005).

This research emphasizes the necessity of integrating shake table dynamics into the sub-structuring test framework. The principal objective of this study is preassessment of RTHST testing scenario. The response of a 10-degree-of-freedom (10-DOF) system subjected to ground motion is chosen for pre-assessment. The equations of motion of PS and NS of 10-story building are formulated in a state-space state. NS, shake table, control system, and PS are numerically formulated in MATLAB Simulink platform. The investigation considers an electrodynamic shake table integrated with a control system to replicate seismic excitations and assess system behaviour accurately. PID controller is incorporated for delay compensation. The performance of RTHSTT simulation is assessed with reference to numerical response of 10 story building. This study offers validation remarks of numerical algorithms and control techniques system to compensate for delays effectively by considering PS as a numerical.

2 FORMULATION FOR RTHST TEST

The equation of motion of 10 story building subjected to earthquake ground motion is,

$$\underline{M}\,\ddot{\underset{\sim}{x}}(t) + \underline{C}\,\dot{\underset{\sim}{x}}(t) + \underline{K}\,\underset{\sim}{x}(t) = -\underline{M}\,\underset{\sim}{r}\,\ddot{\underset{\sim}{x}}_g(t) \tag{1}$$

where $\underline{M}$, $\underline{C}$, and $\underline{K}$ are mass, damping, and stiffness matrices of size 10 x 10. $\ddot{x}(t)$, $\dot{x}(t)$ and x(t) are relative acceleration, velocity, and displacement vectors of 10 DOFs. r, rigid body vector, is the size 10 x 1 of unity vector. $\ddot{xg}(t)$ is acceleration of earthquake ground motion.

2.1 Substructure Modelling

For RTHST test, 10 story building is divided into two parts viz. numerical substructure (NS) and physical substructure (PS). The first to eight stories of the building is considered as NS, and ninth and tenth story of the building is considered as PS. Thus, NS has 8 DoFs and PS has 2 DoFs. The schematics of 10 story building along with NS and PS is shown in Figure 1. In the substructing method, the response of NS and PS is interrelated (Lee et al., 2007). The NS is analyzed for a combination of earthquake ground motion and reaction from PS. PS is subjected to a total acceleration of top (eighth) floor of NS. Division of 10 DoF and equation of motion of NS and PS is formulated as,

$$\begin{bmatrix} m_1 & & & & \\ & \ddots & & & \\ & & m_8 & & \\ & & & m_9 & \\ & & & & m_{10} \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \vdots \\ \ddot{x}_8 \\ \ddot{x}_9 \\ \ddot{x}_{10} \end{Bmatrix} + \begin{bmatrix} c_1+c_2 & -c_2 & & & \\ \ddots & \ddots & \ddots & & \\ & -c_7 & c_8+c_9 & -c_9 & \\ & & -c_9 & c_9+c_{10} & -c_{10} \\ & & & -c_{10} & c_{10} \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \vdots \\ \dot{x}_8 \\ \dot{x}_9 \\ \dot{x}_{10} \end{Bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 & & & \\ \ddots & \ddots & \ddots & & \\ & -k_7 & k_8+k_9 & -k_9 & \\ & & -k_9 & k_9+k_{10} & -k_{10} \\ & & & -k_{10} & k_{10} \end{bmatrix} \begin{Bmatrix} x_1 \\ \vdots \\ x_8 \\ x_9 \\ x_{10} \end{Bmatrix} = -[M]\{r\}\{\ddot{x}_g\} \quad (2)$$

NS
PS

The equation of motion of numerical and physical sub-structures are as follows

$$\underline{M}_N \ddot{\underset{\sim}{x}}_N + \underline{C}_N \dot{\underset{\sim}{x}}_N + \underline{K}_N \underset{\sim}{x}_N = -\underline{M}_N \underset{\sim}{r} \ddot{\underset{\sim}{x}}_g + \underset{\sim}{i}(t) \quad (3)$$

where, $\{i(t)\}_{8x1} = \{\{0\}_{7x1} \quad c_9(\dot{x}_9-\dot{x}_8)+k_9(x_9-x_8)\}^T_{8x1}$

$$\underline{M}_E \ddot{\underset{\sim}{x}}_E + \underline{C}_E \dot{\underset{\sim}{x}}_E + \underline{K}_E \underset{\sim}{x}_E = -\underline{M}_E \underset{\sim}{r} \ddot{\underset{\sim}{x}}_g + \underset{\sim}{q}(t) \quad (4)$$

Where, $\{q(t)\}_{2x1} = \{k_9 x_8 + c_9 \dot{x}_8 \quad 0\}^T_{2x1}$

where subscript N and E represent the numerical and physical sub-structures, respectively. i(t) and q(t) vectors termed as the feedback force to NS and input force to PS. In the present study, the responses of both NS and PS are assumed to be linear and elastic.

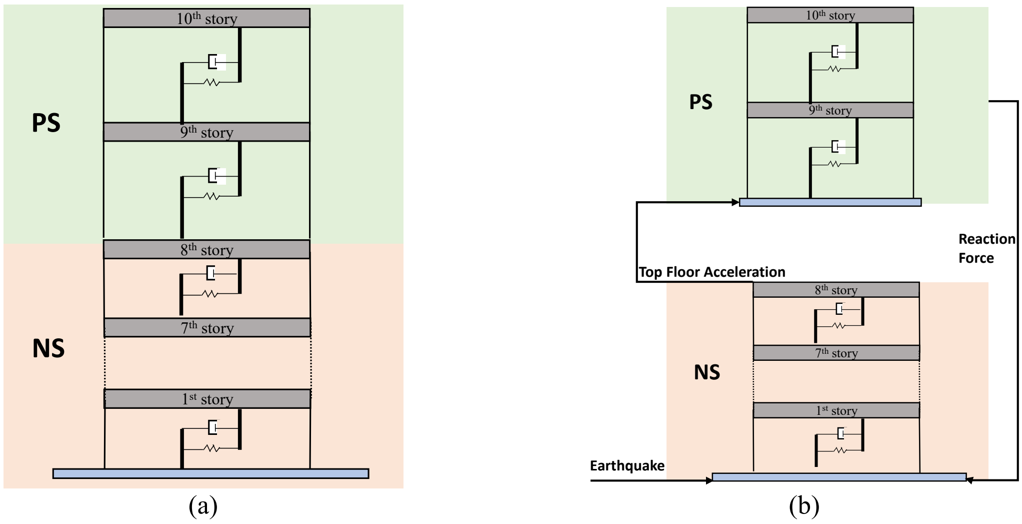


Figure 1: (a) Complete system and (b) Sub-structuring system

For effective modelling of RTHST, state-space form (Shrikhande 2008) is employed to describe the motion of NS and PS. The state-space form for NS is defined with state vector $z_N(t)$ and input and output response vectors $u_N(t)$, and $O_N(t)$, respectively. The state space equations for NS are as

State space vector, $\{z_N(t)\} = \{\{\underset{\sim}{x}_N\}_{8x1} \quad \{\dot{\underset{\sim}{x}}_N\}_{8x1}\}$

Input response vector, $\{u_N(t)\} = \{\ddot{x}_g \quad i(t)\}^T$

Output response vector, $\{O_N(t)\} = \{x_8 \quad \dot{x}_8\}^T$

$$\{\dot{z}_N(t)\}=[A_N]\{z_N(t)\}+[B_N]\{u_N(t)\}$$

$$\{O_N(t)\}=[C_N]\{z_N(t)\}+[D_N]\{u_N(t)\} \tag{5}$$

where A_N, B_N, C_N, and D_N are matrix coefficients as,

$$[A_N]=\begin{bmatrix} [0]_{8x8} & [I]_{8x8} \\ -\left[[M_N]^{-1}[K_N]\right]_{8x8} & -\left[[M_N]^{-1}[C_N]\right]_{8x8} \end{bmatrix}_{16x16},\ [B_N]=\begin{bmatrix} \{0\}_{8x1} & \{0\}_{8x1} \\ \{-1\}_{8x1} & [M_N]^{-1}\left\{\{0\}_{7x1}\{1\}\right\}^T \end{bmatrix}$$

$$[C_N]=\begin{bmatrix} [0]_{7x1} & [1]_{1x1} & [0]_{8x1} \\ [0]_{8x1} & [0]_{7x1} & [1]_{1x1} \end{bmatrix}_{16x2},\ [D_N]=[0]_{2x2}$$

Similarly, the state space equations for PS are as follows:

State space vector, $\{z_E(t)\}=\{\{\underset{\sim}{x}_E\}_{2x1} \quad \{\dot{\underset{\sim}{x}}_E\}_{2x1}\}$

Input response vector, $\{u_E(t)\}=\{\ddot{x}_g \quad q(t)\}^T$

Output response vector, $\{O_E(t)\}=\{x_9 \quad \dot{x}_9 \quad \ddot{x}_9 \quad \ddot{x}_{10}\}^T$

$$\{\dot{z}_E(t)\}=[A_E]\{z_E(t)\}+[B_E]\{u_E(t)\} \tag{6}$$

$$\{O_E(t)\}=[C_E]\{z_E(t)\}+[D_E]\{u_E(t)\}$$

where A_E, B_E, C_E, and D_E are matrix coefficients as,

$$[A_E]=\begin{bmatrix} [0]_{2x2} & [I]_{2x2} \\ \left[-[M_E]^{-1}[K_E]\right]_{2x2} & \left[-[M_E]^{-1}[C_E]\right]_{2x2} \end{bmatrix},\ [B_E]=\begin{bmatrix} \{0\}_{2x1} & \{0\}_{2x1} \\ \{-1\}_{2x1} & [M_E]^{-1}\{1,0\}^T \end{bmatrix}$$

$$[C_E]=\begin{bmatrix} [1,0] & [0]_{1x2} \\ [0]_{1x2} & [1,0] \\ \left[-[M_E]^{-1}[K_E]\right]_{2x2} & \left[-[M_E]^{-1}[C_E]\right]_{2x2} \end{bmatrix},\ [D_E]=[B_E]$$

2.2 Modelling with Shake Table Dynamics and Compensation Framework

In RTHST, the response of NS is applied to the PS, through the shake table. For accurate simulation of RTHST, the shake-table dynamics incorporated in the system model. The force required for the motion of shake table is generated by the coil placed over the armature, i.e. electromagnetic force. This force is equal to the multiplication of magnetic flux, current passing through the coil, and length of the wire within flux. The motion of armaature is controlled using PID controller. The PID controller also serves for the delay compensation and synchronization between NS and PS (discussed detail in section 3.2.2). The schematic of electro-dynamic shake table with PID controller is shown in *Figure 2*.

Verma et al., (2019) provided equation of motion for the electro-dynamic shake table. The same is used in the present study, is as follows.

$$M_a\ddot{x}_t+K_a x_t+C_a\dot{x}_t=Bil \tag{7}$$

where M_a, C_a, and K_a is the mass, damping, and stiffness of armature moving of the shake table, respectively.

B, i, and l are magnetic flux density, current and length of coil, respectively.

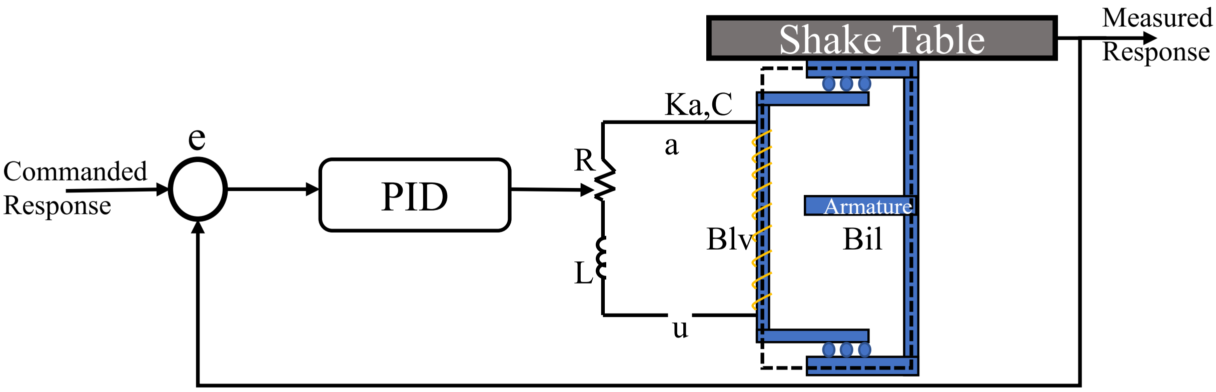


Figure 2: Schematic of an electrodynamic Shake table with PID Control

The relative motion of the armature and magnetic field induces back electro-magnetic force and, which is opposed by externally applied voltage. Kirchhoff's law is applied to maintain the equilibrium in the circuit, as shown in equation (8).

$$G_v u - Bl\dot{x}_t = Ri + L\frac{di}{dt} \tag{8}$$

L, G_v, and R are inductance, voltage gain, and resistance of the electrodynamic shake table. The state space equations for the electro-dynamic shake table are as follows:

State space vector, $\{Z_s(t)\} = \{x_t \quad \dot{x}_t \quad i\}$

$$\{\dot{z}_s(t)\} = [A_s]\{z_s(t)\} + [B_s]\{u_s(t)\} \tag{9}$$

$$\{O_s(t)\} = [C_s]\{z_s(t)\} + [D_s]\{u_s(t)\}$$

where A_S, B_S, C_S, and D_S are matrix coefficients as,

$$[A_s] = \begin{bmatrix} 0 & 1 & 0 \\ -K_a/M_a & -C_a/M_a & Bl/M_a \\ 0 & -B & -R/L \end{bmatrix}, \ [B_s] = \begin{bmatrix} 0 \\ 0 \\ G_v/L \end{bmatrix}$$

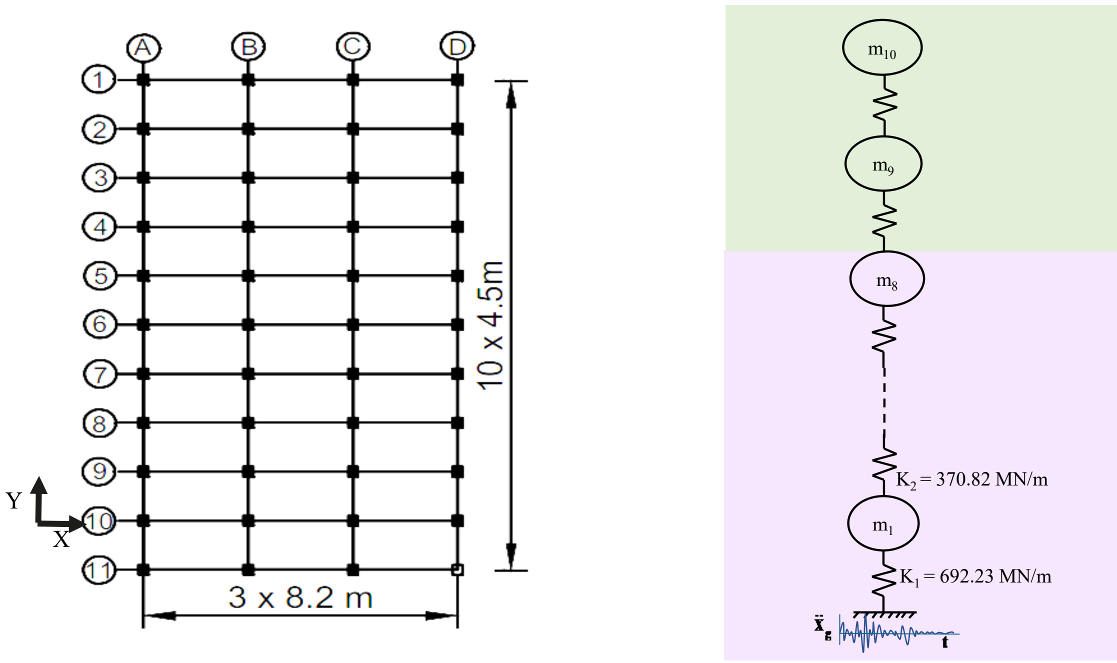
$$[C_s] = [-K_a/M_a \quad -C_a/M_a \quad Bl/M_a], \ [D] = [0]$$

3 IMPLEMENTATION OF RTHSTT FOR MDOF SYSTEM

A 10-DOF structural system is designed in ETABS to evaluate the physical parameters of the system. The seismic response of the system is analysed using the Newmark-beta method in MATLAB. The structure is divided into an 8-DOF NS and a 2-DOF PS considered as numerical modelled in Simulink with shake table dynamics.

3.1 MDOF System

A 10 story Moment resistant braced steel frame building with RC slab is considered in the present study. The building is designed as per current Indian standards. The dead load, superimposed dead load, and live load are considered in the design as per IS 875 part I and II. The design seismic load is estimated as per IS 1893 (2016). The building is assumed to be located in seismic zone IV on medium soil. The analytical model of the building is analyzed for all the loads, and design of structural members is carried out as per IS 800 (2007). The building plan along with lump mass model, are shown in *Figure 3*. The stiffness and mass of each DoF (story) is also shown in Figure 3(b).



(a) (b)

Figure 3: (a) Plan view of 10 story building (b) Lumped-mass Model of 10-Story MR Frame Building

3.1.1 LTH of MDoF

For linear time history (LTH) analysis i.e. linear seismic response analysis of MDOF system, a time-marching Newmark-beta method is implemented in MATLAB (Chopra, 2007). This MATLAB code is used to determine the response of NS. In the present study, to demonstrate the implementation of RTHST framework, El Alamo, 1956 earthquake ground motion (PGA= 0.0334g) is considered the input to the base of the building.

3.2 Synchronization of PS and NS with Shake-Table Dynamics

The synchronization NS and PS with shake-table dynamics is shown in *Figure 4*. The sub-structuring Simulink model is divided into three primary components: NS, shake-table dynamics with PID control, and PS. Each component and its role in the system are explained in detail below.

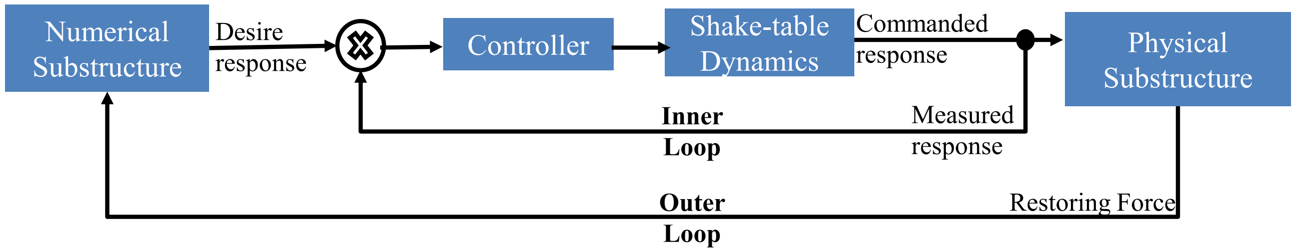


Figure 4: Schematic diagram of PS -NS synchronization adopted in Simulink for RTHST

In the RTHST, the system is designed with a dual-loop architecture. The inner loop tracks that the shake table motion closely is as per desired response of NS. The outer loop manages the synchronization between the PS and NS. This loop ensures seamless interaction between the two components, enabling the hybrid system to behave as a complete structure.

3.2.1 Numerical Sub-Structure Module

The NS component of Simulink model framework is shown in *Figure 5*. The input for NS are earthquake ground motion, denoted as the "Ground Motion" block and force feedback from PS" Interaction Force (i(t))".

The input vectors feed into state-space equation block to get the displacement and velocity responses of 8th story (equation 5). The interaction force to PS i.e. q(t) is estimated from responses of 8th story (equation 4). The interaction force is applied to PS through an electro-dynamic shake table. In the simulated system, responses of 8th story are transferred to PS module to calculate interactive force from PS to NS (discussed further in section 3.3.3).

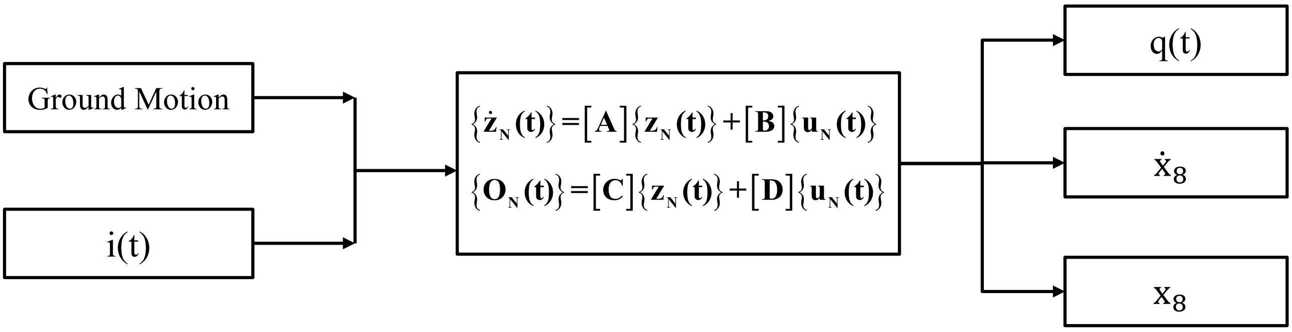


Figure 5: Schematic diagram of Numerical Sub-structure Simulink Model

3.2.2 PID module

The PID module is critical in RTHST. The role of PID module is twofold. First, PID needs to ensure that the motion of shake table is as per the input from NS i.e. inner loop control. Additionally, PID needs to compensate for the delay due to shake table dynamics i.e. outer loop control. Thus shake table dynamics is also incorporated to in this module to simulate the motion and delay of the shake table. Simulink model of shake table dynamics with PID controller as shown in *Figure 6.*

The motion of shake table (with PS) is incorporated in terms of the Transfer function. The transfer function is developed by solving governing equation of shake table dynamics (equation 9) for physical parameters of electro-dynamic shake table. The physical parameters of electro-dynamic shake table given by (Verma et al., 2019) are used in the present study. The parameters are summarised in Table 1. The transfer function for shake table with PS is as follows:

$$TF=\frac{-356.73s^2}{s^3+6.818s^2+256.6s+0.001167} \tag{10}$$

The input to PID module is the force (q(t)) from the NS to PID Controller. The PID controller incorporates proportional, integral, and derivative control actions to maintain desired motion of shake table as well as to compensate for the delay in response of PS.

PID module available in MATLAB Simulink (Visioli, 2003) is used for simulation. The gain parameters of PID controller are determined from the calibration complete RTHST system. The calibration is based on system identification of the complete RTHST system i.e. simultaneous control of inner and outer loops. Auto tunning option in MATLAB Simulink is used to determine gain parameters of PID controller. For the complete RTHST system, the identified values of gain parameters K_p, K_i, and K_d are are – 0.259e3, -4.45e4, and 1.03e-5, respectively.

Table 1: Physical parameter of Electro-dynamic Shake-table

Parameter	Value
Force to current ratio (Bl)	19.62
Mass of the shaker(ma), kg	17.9
Damping in voltage mode(c), Ns/m	25

Stiffness of the shaker (Ka), N/m	175
Coil inductance(L), H	0.22
Coil Resistance (R), Ω	1.5
Amplifier gain (G_v)	-4

Simultaneously, a feedback loop is closed to the PID controller to continually adjust the shake table's operation, ensuring accurate tracking of the NS command, this termed inner loop control and the response synchronization.

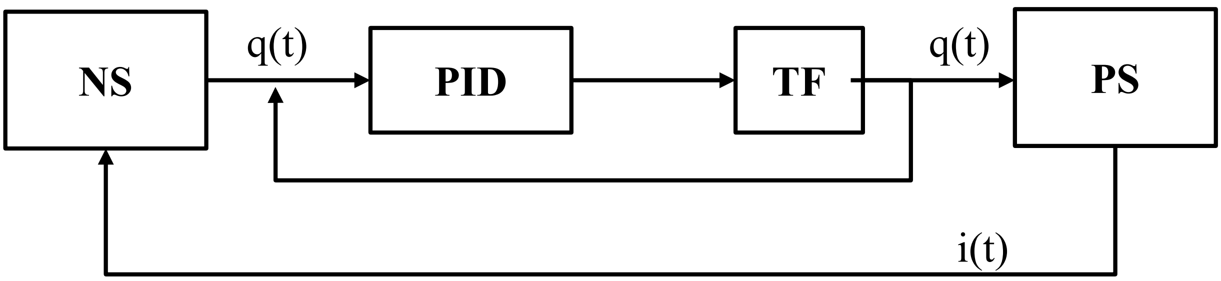


Figure 6: Schematic diagram Shake-Table Dynamics with PID Controller Simulink model

3.2.3 Physical Sub-Structure Module

PS module comprises of motion of PS and interaction force from PS to NS. The PS component of Simulink component is shown in Figure 7. The shake table force and ground motion are inputed to state-space equation of PS (equation 6). The interactive force (i(t)) from PS to NS is estimated from displacement and velocity at 9th story, obtained from state-space equation of PS, and displacement and velocity 8th story (obtained from NS). The interaction force is fed back to the NS.

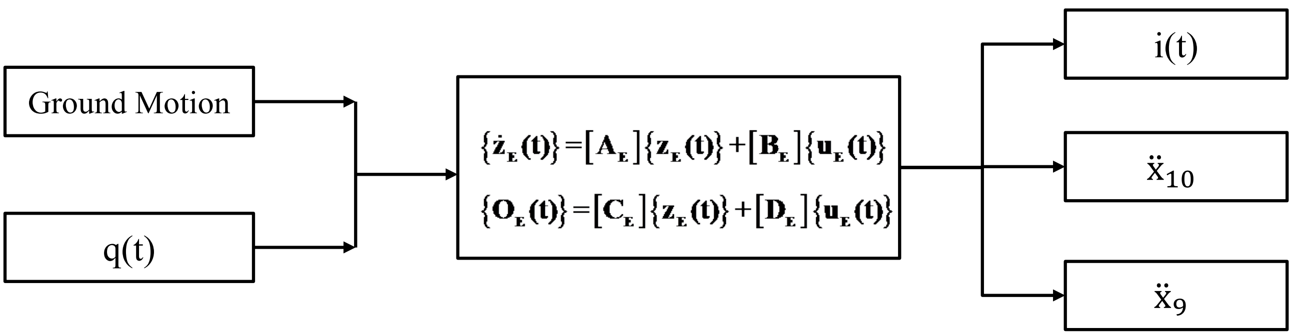


Figure 7: Schematic diagram of Physical Sub-structure Simulink Model

4 RESULTS

4.1 Motion of Shake Table

The role of PID control is to maintain the motion of the shake table as per the input signal from NS. Figure 8 shows input signal from NS and motion of shake table considering shake table dynamics. There is complete match between input signal from NS and simulated motion of the shake table. Thus, the developed PID module effectively controls the inner loop of shake table.

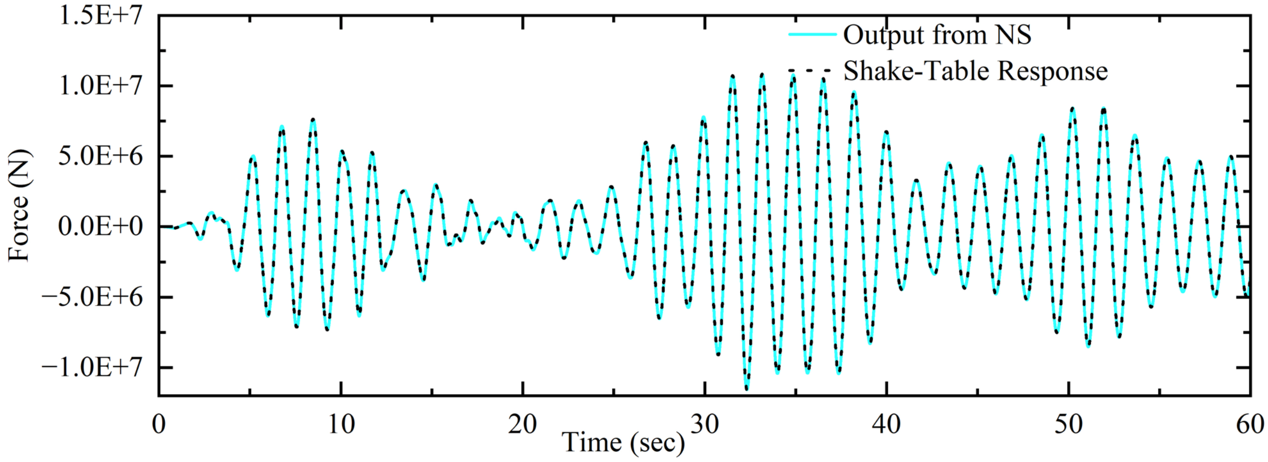


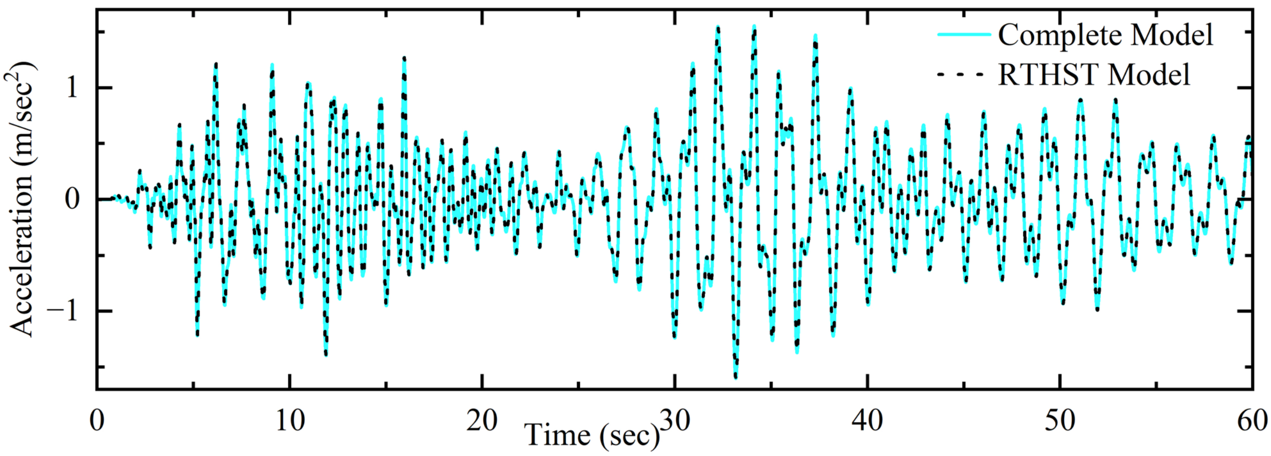
Figure 8: Input from NS and Output from Shake-table

4.2 Validation of RTHST

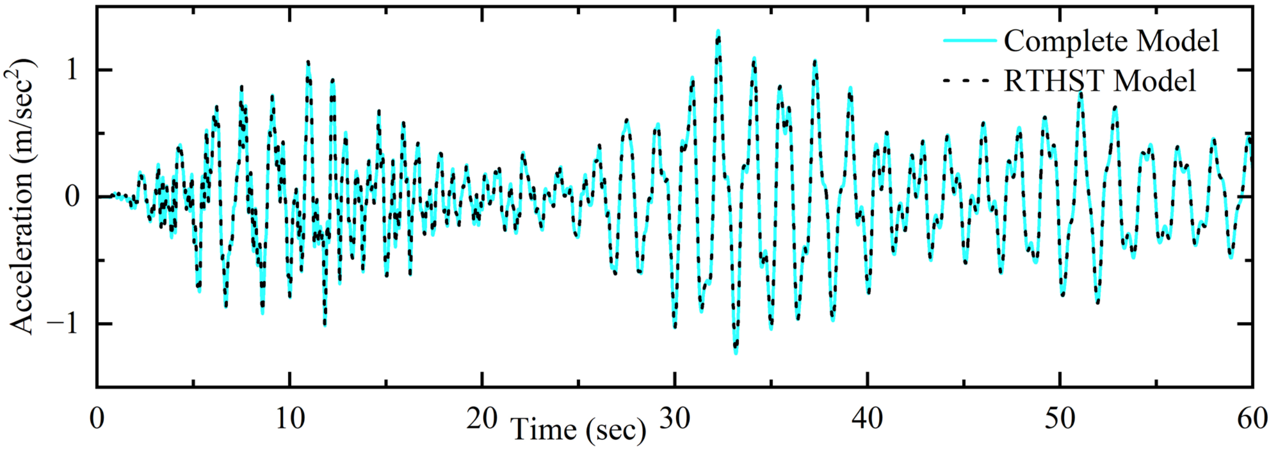
The developed simulation of RTHST is validated by comparing the responses from PS of RTHST module those from the complete system. Linear time history analyses of complete 10 DoF system is carried using the Newmark-beta method in MATLAB. The LTH is carried out for same earthquake ground motion. Figure 9 compares relative accelerations of 9th and 10th stories obtained from RTHST system and LTH analysis. A complete match is observed between accelerations from RTHST system simulation and LTH analysis. The peak response (displacement, velocity and acceleration) of 9th and 10th stories obtained from RTHST system and LTH analysis are approximately equal as shown in Table 2.

Table 2: PGA, PGV and PGD of complete system and RTHST

Parameter	Complete System		RTHST	
	9th Story	10th Story	9th Story	10th Story
PGD(m)	0.068	0.0739	0.0679	0.0728
PGV(m/s)	0.27	0.317	0.265	0.309
PGA(g)	0.1337	0.159	0.133	0.158



(a)



(b)

Figure 9: Acceleration Response of complete system and RTHST a)10th Floor of the Structure b) 9th Floor of the Structure

5 SUMMARY

This study assesses a 10-story building subjected to earthquake ground motion, where the equations of motion for PS and NS are formulated in a state-space framework and implemented in MATLAB Simulink. RTHST is more effective for assessing structural behavior under earthquake loading. This paper provides the detailed framework of each component, i.e., NS, PS, and shake table, with the controller implied in RTHST. This paper gives a brief of RTHST to validate the numerical algorithm and mitigate the delay arising from computational processing and shake table dynamics with PID controller. Additionally, the research needs to be explored for validation of numerical algorithms and developing delay compensation techniques for nonlinear structural system.

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