

Earthquake design of retaining walls using a refinement to limit state analysis

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ABSTRACT

Free-standing retaining walls subjected to earthquake loading are commonly designed using the Mononobe–Okabe (M–O) method, which assumes a linear distribution of combined gravity and seismic earth pressures with the resultant acting at one-third of the wall height above the base. Although this assumption is convenient, it does not in general satisfy moment equilibrium of the failure wedge and can lead to unconservative estimates of the moment demand on the wall, particularly for walls with low wall–soil interface friction, sloping backfills, or when designed for strong seismic excitation.

This paper presents a refined limit-state analysis based on the method originally proposed by Prakash and Basavanna (1969), reformulated to provide a computationally efficient framework that rigorously satisfies force and moment equilibrium of the M–O failure wedge. The method is extended to include, backfill cohesion, surcharge loading, and layered soil profiles.

Parametric analyses demonstrate that the centre of pressure for combined gravity and earthquake loading frequently lies well above $0.33H$, exceeding $0.45H$ for smooth walls under moderate seismic loading and exceeding $0.7H$ for walls with low wall interface friction and sloping backfills. A multi-layer formulation is also presented to approximate the pressure distribution over the wall height when more than a single resultant force is required.

The results show that while simplified assumptions may be acceptable for low walls under moderate seismic loading, more accurate estimation of the force location using the refined Prakash and Basavanna approach is necessary to avoid unconservative design in practical situations involving major walls.

1 INTRODUCTION

Many low walls are of cantilever type or embedded pile construction. In this type of wall, lateral pressures from gravity and horizontal earthquake forces will usually produce sufficient displacement within the wall structure and foundation to induce a fully plastic stress state in the retained soil. In more rigid free-standing walls, such as gravity (e.g. mechanically stabilised earth and concrete block walls) counterfort walls and building basement walls, a fully plastic stress state may develop as the result of permanent outward movement from sliding or rotational deformations in the foundation. In cases where significant nonlinear soil behaviour or a fully plastic stress state occurs in the backfill soil during earthquake loading, the well-

known Mononobe-Okabe (M-O) analysis method (Mononobe and Matsuo 1929) can be used to compute the combined earthquake and gravity force from the backfill on the wall. This method is an extension of the Coulomb (1776) analysis for gravity backfill loads on retaining walls.

In routine application of the M–O method, it is commonly assumed that the combined gravity and earthquake-induced earth pressures vary linearly with depth, resulting in a single resultant force acting at one-third of the wall height above the base. This assumption is frequently adopted irrespective of wall friction, backfill geometry, or seismic intensity. However, it has long been recognised that this simplification is not, in general, consistent with the full equilibrium conditions of the soil failure wedge. Terzaghi (1941) showed that when wall–soil interface friction is significant, a linear pressure distribution does not satisfy moment equilibrium under gravity loading alone. Similar inconsistencies arise under combined gravity and seismic loading.

Prakash and Basavanna (1969) presented an important but largely overlooked refinement to Coulomb-type analyses by explicitly enforcing force and moment equilibrium of the failure wedge. Their method demonstrated that the location of the resultant earth pressure depends on, wall friction, backfill geometry, and level of seismic loading and does not necessarily correspond to one-third of the wall height. Despite its sound theoretical basis, the method has seen limited adoption in practice, largely because of its algebraic complexity and the restricted range of parameters examined in the original study.

Experimental investigations using shaking tables and centrifuge testing have provided mixed but instructive evidence regarding the magnitude and location of seismic earth pressures. While many studies confirm that the total thrust predicted by the M–O method is reasonable once sufficient wall displacement occurs, several investigators have observed that the dynamic component of the pressure acts significantly higher than the $0.33H$ assumed in conventional design. These observations suggest that inaccuracies in the assumed point of application, rather than in the magnitude of the thrust, may result in unconservative estimates of critical wall response modes such as overturning and bearing pressure failure.

The present paper revisits and extends the Prakash and Basavanna approach with the objective of making it both more general and more accessible for modern seismic retaining wall analysis. The method uses a two-stage equilibrium and superposition procedure that leads to a computationally efficient solution while rigorously satisfying horizontal force, vertical force, and moment equilibrium. The analysis is extended to include backfill cohesion, wall adhesion and backfill surcharge loading, while retaining compatibility with generalized M–O solutions for total thrust magnitude that include these backfill factors. In addition to determining the resultant force and its height of application, a multi-layer formulation is developed to approximate the distribution of combined gravity and seismic pressures over the wall height.

The primary focus of the paper is to quantify the conditions under which the conventional assumption of a resultant acting at $0.33H$ becomes unconservative, and to provide practical guidance on when a refined equilibrium-based analysis is warranted. The results are intended to complement, rather than replace, existing pseudo-static design methods by improving the estimation of force location while retaining familiar assumptions regarding failure mechanisms and soil behaviour.

2 PRAKASH AND BASAVANNA ANALYSIS

2.1 Underlying Assumptions

The underlying assumptions made by Prakash and Basavanna in their failure wedge analysis are:

1. The backfill soil material in the failure wedge is cohesionless.
2. The rupture surface is plane.
3. The shear resistance along this plane is fully mobilised.

4. The vertical pressure on a plane parallel to the ground surface is constant.
5. Equations of equilibrium for horizontal and vertical forces and moment about any point are satisfied.
6. The principle of superposition can be applied.

2.2 Analysis Method

The analysis definition diagram is shown in Figure 1. The analysis is based on rotating the principal xx and yy axes with origin at the bottom of the wall backface in an anticlockwise direction by the angle of inclination β of the surface of the backfill. The rotated axes are labelled x'x' and y'y'. The forces acting on the failure wedge and the triangular area to the left of axis yy formed by the axis rotation are:

W_1 , the weight of ADC which includes the failure wedge and the triangle to the left of the principal axis acting vertically downwards at distances of $2/3AD$ and $1/3DC$ from point A on the rotated axes $x'x'$ and $y'y'$ respectively.

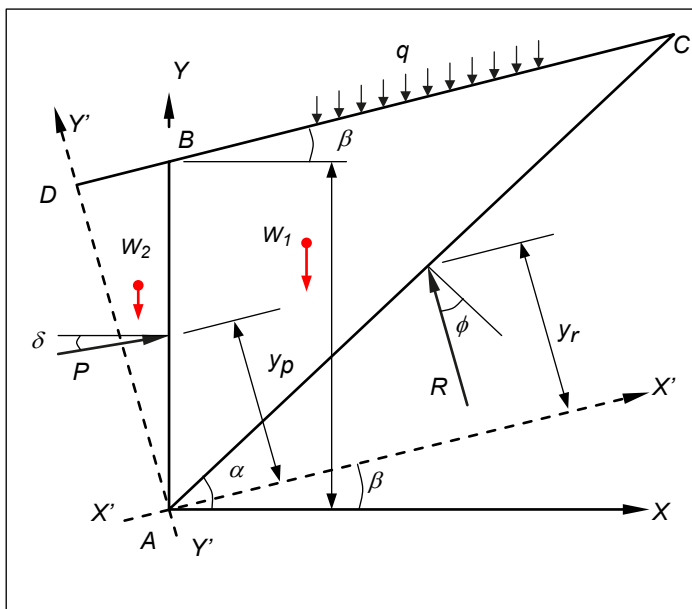
W_2 , the weight of the triangle ADB located to the left of the principal axis yy and acting vertically downwards at distances of $2/3AD$ and $1/3 DB$ from point A on the rotated axes $x'x'$ and $y'y'$ respectively.

P , the active pressure force acting on the retaining wall inclined at an angle $(\delta - \beta)$ to the $y'y'$ axis at a perpendicular distance y_p from the $x'x'$ axis.

R , the reaction force on the failure plane AC inclined at an angle $(\alpha-\phi-\beta)$ to the $y'y'$ axis at a perpendicular distance y_r from the $x'x'$ axis.

$W_1 k_v$ and $W_2 k_v$, inertia forces acting vertically and located at the W_1 and W_2 centres of weight respectively.

$W_1 k_h$ and $W_2 k_h$, inertia forces acting horizontally in a negative xx direction and located at the W_1 and W_2 centres of weight respectively.



The symbols used in the equations presented below are:

γ the bulk unit weight of the backfill soil, δ the angle of wall-soil interface friction, ϕ the angle of internal friction of the backfill soil, β the slope of the backfill surface, α the slope of the failure surface measured from the horizontal, q the superposed load on the backfill surface per unit sloping length, c the average cohesion per unit sloping length on the failure surface and c_a the ratio of the adhesion force on the wall-soil interface per unit length to the cohesion on the failure surface per unit sloping length, k_h and k_v the horizontal and vertical acceleration coefficients respectively, and H the height of the wall.

Figure 1. Definition diagram for Prakash and Basavanna analysis.

The cohesion and adhesion were assumed to be uniform over the failure surface length and the wall-soil interface respectively and no tension crack depth was defined but its influence can be included by adjusting the cohesion to be a reduced average value.

The forces and weights have been expressed in dimensionless form by dividing them by $\gamma H^2/2$ and the moments by dividing them $\gamma H^3/6$.

The dimensionless weights of the areas associated with the failure surface are:

$$W_1 = \cos^2 \beta \cot(\alpha - \beta) \quad (1)$$

$$W_2 = -\cos \beta \sin \beta \quad (2)$$

$$W_t = W_1 + W_2 \quad (3)$$

The dimensionless surcharge forces Q_1 and Q_2 acting on the surface lengths DC and DB of the triangular areas ADC and ADB respectively are:

$$Q_1 = 2q \cos \beta \cot(\alpha - \beta) / \gamma H \quad (4)$$

$$Q_2 = -2q \sin \beta / \gamma H \quad (5)$$

$$Q_t = Q_1 + Q_2 \quad (6)$$

The cohesive force acting on the failure slope is the cohesion c multiplied by the slope length and in dimensionless form is given by:

$$C_d = 2\bar{c} \cos \beta / \sin(\alpha - \beta) \quad (7)$$

$$\text{Where } \bar{c} = \frac{c}{\gamma H}$$

The adhesion force acting on the wall-soil interface is $c_a c H$ and in dimensionless form is given by:

$$C_{ad} = c_a \bar{c} \quad (8)$$

The unknowns in the failure wedge analysis are the active pressure force on the retaining wall, the reaction on the failure plane and the heights at which these two forces act. There are only three equations of equilibrium and with four unknowns there is no direct solution. To solve this problem Prakash and Basavanna used a two-stage analysis and then superposition to combine the two sets of results. In the first stage only the body forces acting in the $x'x'$ direction were considered and in the second stage only the body forces acting in the $y'y'$ direction were applied. In each of these two stages they showed that the perpendicular height of the wall pressure force and the failure surface reaction above the $x'x'$ axis were equal reducing the unknowns in each of the two stages to three parameters that can be evaluated using the conventional equilibrium equations (horizontal and vertical force equilibrium, and moments about a point in the analysis plane.)

In the analyses equations the boundary force components are taken as positive when in the positive direction of the transformed axes. The body force components are taken as positive when in the negative direction of the transformed axes. The signs used in the formulae are based on the assumption that $\delta > \beta$.

2.3 Stage 1 Analysis

In the Stage 1 analysis only the components of the body forces acting in a direction parallel to the $x'x'$ axis are considered. These forces include components of the weight forces, the vertical and horizontal earthquake inertia forces, the cohesion and adhesion forces acting on the vertical and sloping boundaries of the failure wedge, the vertical surcharge load on the backfill surface and the horizontal and vertical inertia forces acting on the surcharge.

In the $y'y'$ direction the only forces are components of P and R . From equilibrium of forces in the $y'y'$ direction:

$$R_1 = -P_1 \frac{\sin(\delta - \beta)}{\cos(\alpha - \beta - \varphi)} \quad (9)$$

Where the subscripts on P_1 and R_1 donate their values in Stage 1.

The sum of the body force components in the $x'x'$ direction is given by:

$$F_1 = (W_t + Q_t)[(1 \pm k_v) \sin \beta + k_h \cos \beta] - \bar{c} \cos \beta [\cot(\alpha - \beta) + c_a \tan \beta] \quad (10)$$

The sum of the boundary force components in the $x'x'$ direction is given by:

$$B_1 = P_1 [\cos(\delta - \beta) + \sin(\delta - \beta) \tan(\alpha - \varphi - \beta)] \quad (11)$$

Where the value of R_1 from Equation 9 has been used.

Equating the body force and boundary forces in the $x'x'$ direction gives:

$$P_1 = \frac{(W_t + Q_t)[(1 \pm k_v) \sin \beta + k_h \cos \beta] - \bar{c} \cos \beta [\cot(\alpha - \beta) + c_a \tan \beta]}{[\cos(\delta - \beta) + \sin(\delta - \beta) \tan(\alpha - \varphi - \beta)]} \quad (12)$$

The heights of the centre of pressures of the forces on the wall and the failure plane are determined by taking moments about Point A at the base of the wall. The moments from the body forces are the moments of the components in the $x'x'$ direction. The moment equilibrium equations can be simplified by considering the components from the body and boundary forces separately. Components are in dimensionless form and are taken as positive in the clockwise direction. The moment from the failure wedge weight and the wedge weight inertia forces is given by:

$$M_{w1} = 2W_t \cos \beta [(1 \pm k_v) \sin \beta + k_h \cos \beta] \quad (13)$$

Where the subscript 1 is used to denote the moment in Stage 1 for the various moment components.

The dimensionless moment from the superimposed load on the backfill and its inertia forces is given by:

$$M_{Q1} = 3Q_t \cos \beta [(1 \pm k_v) \sin \beta + k_h \cos \beta] \quad (14)$$

The moment from the combined adhesion force on the wall face and the cohesion on the failure surface is given by:

$$M_{C1} = 1.5 \bar{c} \cos \beta \sin \beta [\cot \beta \cot(\alpha - \beta) + c_a] \quad (15)$$

The moment from the pressure force against the wall is given by:

$$M_{P1} = P_1 y_{p1} [(\cos(\delta - \beta) + \sin(\delta - \beta) \tan \beta)] \quad (16)$$

Using Equation 9 the moment from the reaction force on the failure slope can be written in terms of P_1 as:

$$M_{R1} = P_1 y_{r1} \sin(\delta - \beta) [\cot(\alpha - \beta) + \tan(\alpha - \varphi - \beta)] \quad (17)$$

From force equilibrium in the $y'y'$ direction of the slice element of the failure wedge shown in Figure 2 that is parallel to $x'x'$, has a thickness d_y , and is located at a depth y_e below the backfill surface the pressures on the wall and failure surface are related by:

$$p_P(y) = r_R(y) \frac{\cos \beta \cos(\alpha - \varphi - \beta)}{\sin(\beta - \delta) \sin(\alpha - \beta)} \quad (18)$$

Where $p_P(y)$ and $r_R(y)$ are the pressures on the wall and failure plane respectively. They are functions of the depth y below the backfill surface.

The depth of the centre of pressure on the wall in the $y'y'$ direction is given by:

$$Y_{\text{PC}} = \frac{\int_0^Y p_P(y) y dy}{\int_0^Y p_P(y) dy} \quad (19)$$

The depth of the centre of the reaction pressure on the failure surface in the $y'y'$ direction is given by:

$$Y_{\text{Rc}} = \frac{\int_0^Y r_R(y)y \, dy}{\int_0^Y r_R(y)dy} \quad (20)$$

Substituting the expression for $p_P(y)$ given in Equation 18 into Equation 19 and comparing with Equation 20 indicates that $Y_{Pc} = Y_{Rc}$ which means that perpendicular heights for both P_1 and R_1 above the $x'x'$ axis are equal, that is $y_{p1} = y_{r1}$.

The height y_{p1} (or y_{r1}) is determined by considering the moment equilibrium of the moment components listed in Equations 13 to 17. The moments M_{p1} and M_{r1} from the boundary forces equals the moments M_{w1} , M_{q1} and M_{c1} from the body forces and this gives:

$$y_{P1} = \frac{M_{w1} + M_{Q1} - M_{C1}}{\bar{M}_{P1} + \bar{M}_{R1}} \quad (21)$$

Where $\bar{M}_{P1}$ and $\bar{M}_{R1}$ are the moment components given by Equations 16 and 17 without the y_{p1} and y_{r1} height factors. The signs of the moment components are determined by the angle parameters but M_{C1} is always opposite in sign to M_{W1} and M_{Q1} .

The vertical height h_{P1} of P_1 from the base of the wall is:

$$h_{p1} = \frac{y_{p1}}{\cos\beta} \quad (22)$$

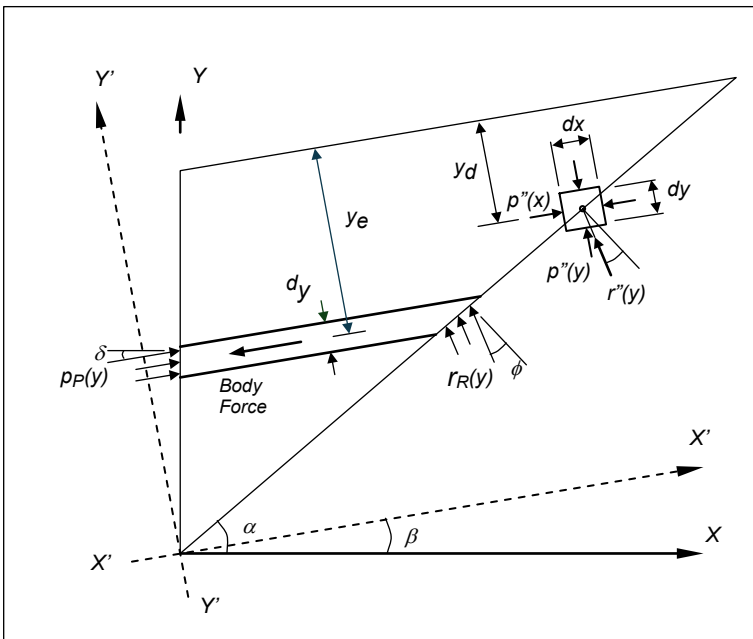


Figure 2. Elements in failure wedge for establishing the relationship between y_p and y_r

2.4 Stage 2 Analysis

In the Stage 2 analysis only the components of the body forces acting in a direction parallel to the $y'y'$ axis are considered. The analysis steps are similar to those of Stage 1 except for the body force directions.

In the $x'x'$ direction the only forces are components of P and R . From equilibrium of forces in the $x'x'$ direction:

$$R_2 = P_2 \frac{\cos(\delta - \beta)}{\sin(\alpha - \varphi - \beta)} \quad (23)$$

Where the subscripts on P_2 and R_2 donate their values in Stage 2.

The sum of the body force components in the $y'y'$ direction is given by:

$$F_2 = (W_t + Q_t)[(1 \pm k_v) \cos \beta - k_h \sin \beta] - \bar{c} \cos \beta (1 + c_a) \quad (24)$$

The sum of the boundary force components in the $x'x'$ direction is given by:

$$B_2 = P_2 [\sin(\delta - \beta) + \cos(\delta - \beta) \cot(\alpha - \varphi - \beta)] \quad (25)$$

Where the value of R_2 from Equation 23 has been used.

Equating the body force and boundary forces in the $y'y'$ direction gives:

$$P_2 = \frac{(W_t + Q_t)[(1 \pm k_v) \sin \beta - k_h \cos \beta] - \bar{c} \cos \beta (1 + c_a)}{[\sin(\delta - \beta) + \cos(\delta - \beta) \cot(\alpha - \varphi - \beta)]} \quad (26)$$

Following the procedure described in Stage 1 the heights of the centre of pressures of the forces on the wall and the failure plane are determined by taking moments about Point A at the base of the wall. The moments from the body forces are the moments of the components in the $y'y'$ direction. The moment from the failure wedge weight and the wedge weight inertia forces is given by:

$$M_{w2} = [W_1 \cot(\alpha - \beta) + W_2 \tan \beta] \cos \beta [(1 \pm k_v) \cos \beta - k_h \sin \beta] \quad (27)$$

Where the subscript 2 is used to denote the moment in Stage 2 for the moment components.

The moment from the superimposed load on the backfill and its inertia forces is given by:

$$M_{Q2} = 1.5[Q_1 \cot(\alpha - \beta) \cos \beta + Q_2 \sin \beta][(1 \pm k_v) \cos \beta - k_h \sin \beta] \quad (28)$$

The moment from the combined adhesion force on the wall face and the cohesion on the failure surface is given by:

$$M_{C2} = 1.5 \bar{c} \cos \beta \sin \beta [\cot \beta \cot(\alpha - \beta) + c_a] \quad (29)$$

The moment from the pressure force against the wall is given by:

$$M_{P2} = P_2 y_{p2} [\cos(\delta - \beta) + \sin(\delta - \beta) \tan \beta] \quad (30)$$

Using Equation 23 the moment from the reaction force on the failure slope can be written in terms of P_2 as:

$$M_{R2} = P_2 y_{r2} \cos(\delta - \beta) [\cot(\alpha - \beta) \cot(\alpha - \varphi - \beta) + 1] \quad (31)$$

The relationship between y_{p2} and y_{r2} can be determined by considering an element of dimensions $dx \times dy$ located on the failure plane of the backfill at a perpendicular depth of y_d below the backfill surface as shown in Figure 2. Normal stresses $p''(x)$ and $p''(y)$ act on the element. The resultant normal stress $r''(y)$ acting on the failure plane can be determined by a Mohr's circle analysis.

From a geometric analysis of the Mohr's circle the normal stress on the failure plane is related to $p''(y)$ by:

$$r''(y) = p''(y) \frac{\cos\phi}{[1 + \sin\phi \sin(2\alpha - 2\beta - \phi)]} \quad (32)$$

$$\text{or } r''(y) = p''(y) K_1$$

The pressure on the wall face can be written as:

$$p_p''(y) = K_a p''(y) \quad (33)$$

Where K_a is the Coulomb active pressure coefficient and is a function of the angle parameters α , β , δ and ϕ and is independent of y . The resultant stress on the failure slope can be written as:

$$r''(y) = K_1 / K_a p_p''(y) \quad (34)$$

The depth of the centre of pressure on the wall in the $y'y'$ direction is given by:

$$Y_{P2c} = \frac{\int_0^Y p_p''(y) y dy}{\int_0^Y p_p''(y) dy} \quad (35)$$

The depth of the centre of the reaction pressure on the failure surface in the $y'y'$ direction is given by:

$$Y_{R2c} = \frac{\int_0^Y r''(y) y dy}{\int_0^Y r''(y) dy} \quad (36)$$

Substituting the expression for $r''(y)$ given in Equation 34 into Equation 36 and comparing with Equation 35 indicates that $Y_{P2c} = Y_{R2c}$ which means that perpendicular heights for both P_2 and R_2 above the $x'x'$ axis are equal, that is $y_{p2} = y_{r2}$.

The height y_{p2} (or y_{r2}) is determined by considering the moment equilibrium of the moment components listed in Equations 27 to 31. The moments from the boundary forces M_{P2} and M_{R2} equals the moment from the body forces M_{W2} , M_{Q2} and M_{C2} and this gives:

$$y_{P2} = \frac{M_{W2} + M_{Q2} - M_{C2}}{\bar{M}_{P2} - \bar{M}_{R2}} \quad (37)$$

Where $\bar{M}_{P2}$ and $\bar{M}_{R2}$ are the moment components given by Equations 30 and 31 without the y_{p2} and y_{r2} height factors. The signs of the moment components are determined by the angle parameters but M_{C1} is always opposite in sign to M_{W1} and M_{Q1} .

The vertical height h_{p2} of P_2 from the base of the wall is:

$$h_{p2} = \frac{y_{p2}}{\cos\beta} \quad (38)$$

2.5 Combining Stage 1 and 2

Applying the principle of superposition the total pressure force acting on the wall is given by:

$$P_t = P_1 + P_2 \quad (39)$$

and the total moment by:

$$M_t = (P_1 h_{p1} + P_2 h_{p2}) \cos\delta \quad (40)$$

The effective height h_{pt} for the total pressure force P_t is given by:

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$$h_{pt} = M_t / P_t \cos \delta \quad (41)$$

The total wall force, total moment on the wall and the effective height of the pressure force are functions of P_1 , P_2 , h_{p1} and h_{p2} listed above in the two-stage analysis equations which can be evaluated if α the angle of inclination of the failure plane is known. The failure plane inclination can be calculated by trial and error to determine the maximum value of P_t the total pressure force on the wall. Trial and error can be conveniently carried out using the Excell Solver add-in. Prakash and Basavanna also used trial and error to determine the maximum wall moment M_t but only the maximum wall force has been considered in the present application on the assumption that wall sliding is likely to be more critical than a moment failure in the wall structure or a bearing failure in the foundation soil at the toe of the wall. However, the procedure to calculate the maximum wall moment is readily undertaken using Solver to maximise M_t given by Equation 40.

2.6 Typical Wall Solutions

Application of the Prakash and Basavanna method was investigated for a typical vertical wall having a height of 5.0 m and retaining both a cohesionless backfill and a backfill with both internal friction and a small amount of cohesion. For both backfills the internal friction angle was taken as 35° and for the cohesion backfill case the average cohesion on the failure surface was taken as 10 kPa and the average adhesion on the wall surface as 5 kPa (adhesion ratio of 0.5). The backfills were assumed to have a unit weight of 20 kN/m^3 . Pressures on the wall from both a horizontal backfill and a backfill with a 12° slope were investigated

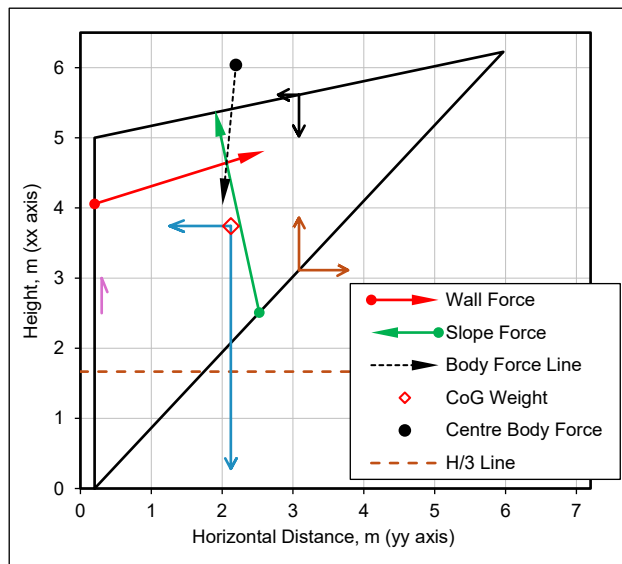


Figure 3. Failure wedge for 5m high wall with c - ϕ backfill and 12° backslope.

The direction of the body force line of action was computed from the ratio of the sum of the horizontal and vertical components of the body forces.

The lines of action of the boundary forces (wall pressure force and failure slope reaction) meet at a common point with the line of action of the body force resultant demonstrating that moment equilibrium of the failure wedge was satisfied.

In Figure 3 the surface superimposed load, failure surface cohesion and wall backface adhesion are shown acting at their centre-points but are in fact uniformly distributed loads over the relevant surfaces.

The magnitude of the forces on the wall and failure surface were calculated using the two-stage analysis procedure and in dimensionless terms were 0.28 and 0.86 respectively. They act at height ratios (actual

The failure wedge and the forces acting on it for the case of the cohesive backfill (, backslope of 12° , surface live load of 5 kPa per slope distance and horizontal acceleration of $0.25 g$ is shown in Figure 3. The centre of the body forces from the failure wedge weight, cohesion on the failure surface, adhesion on the wall backface, superimposed surface load and earthquake forces on both the wedge weight and superimposed load is indicated on the plot at a point 6.04 m above the base of the wall and 2.0 m horizontally from the wall face. This point was calculated by requiring the resultant of the body forces to have the same moment about the base of the wall as the sum of the individual body forces.

height divided by wall height, h_{pt}/H and h_{rt}/H) of 0.81 and 0.50 respectively. The location of the pressure force on the wall is very high and this high value resulted partly from the cohesion in the backfill. For a cohesionless backfill (internal friction of 35° , and wall interface friction of 17.5°) the height ratio reduces to 0.68. The height of the pressure force is also increased by earthquake loading and the backfill surface slope.

Figures 4 (a) and (b) show the wall pressure force height ratios for variations in earthquake acceleration (walls are usually designed for $k_h = 0.5$ of the peak ground acceleration) and wall friction angle respectively. The plotted results show that the height ratio for horizontal backfill surfaces has only a moderate variation of between 0.33 to 0.52 over the range of earthquake accelerations and wall friction angles investigated. Variations in the height ratios are more significant for the sloping backfill case with a variation of 0.28 to 1.0. High values occur for combination of low friction angles and high accelerations. For high friction angles (close to 35°) and for acceleration coefficients greater than 0.25 the height ratio becomes lower than 0.33 but this low value is unlikely in practical applications.

Figures 5 (a) and (b) show the variation with horizontal earthquake acceleration of the wall horizontal pressure force and the force height ratio respectively for a cohesionless backfill with a horizontal surface and internal friction angles of both 30° and 35° . Results are shown for wall backface friction angles of zero, one-half of the soil friction angle, and equal to the soil friction angle. The pressure force height ratios are sensitive to the wall backface friction but are not sensitive to the backfill friction angle.

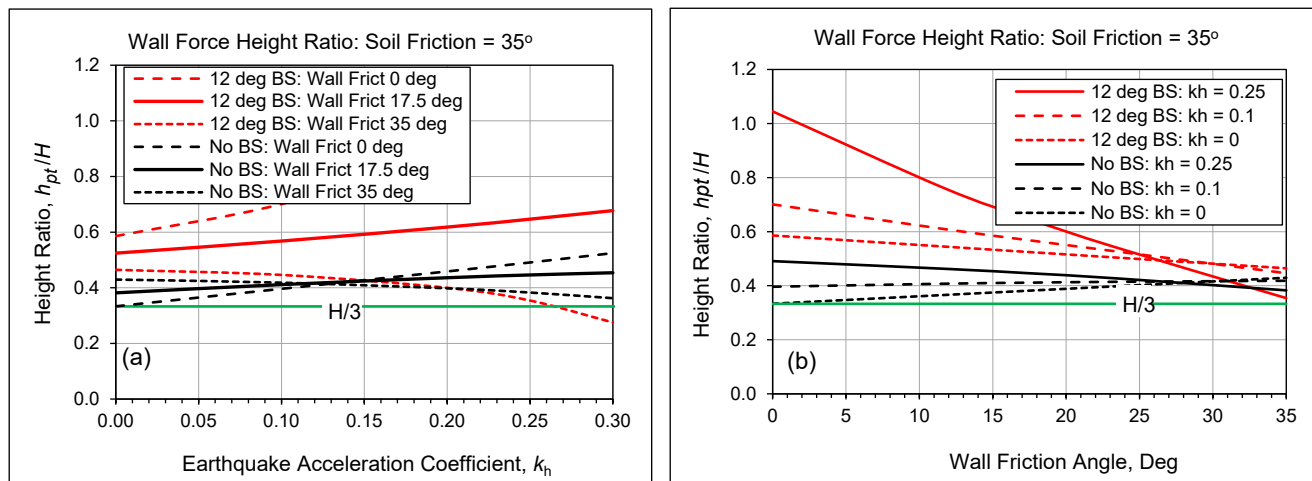


Figure 4. Variation of wall force height ratio: (a) with acceleration coefficient, (b) with wall friction angle.

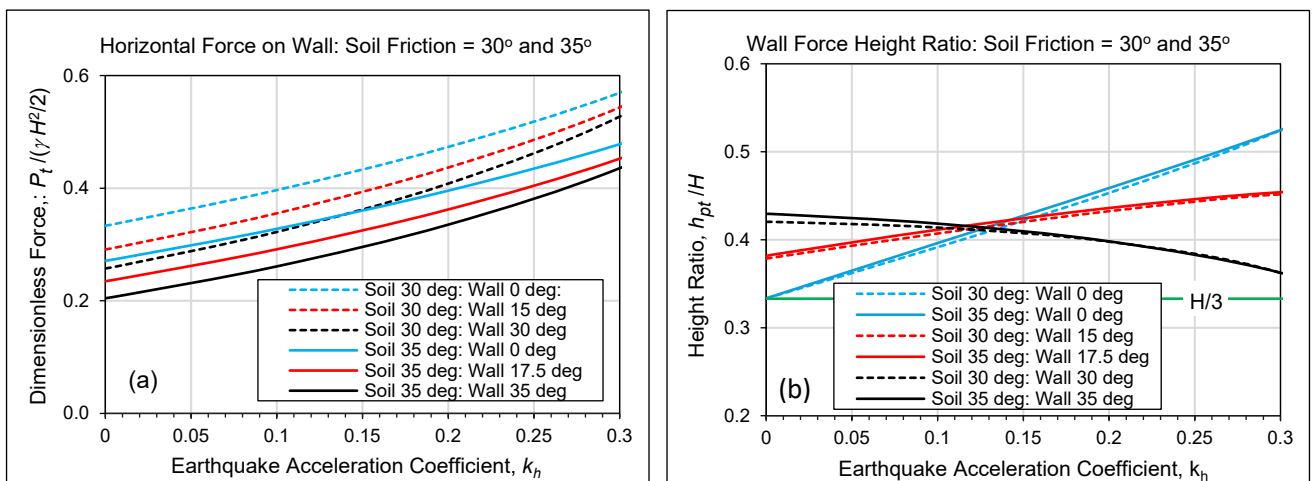


Figure 5. Variation of; (a) horizontal wall force, (b) wall force height ratio with acceleration coefficient.

3 MULTI-LAYER ANALYSIS

In the present study equations were developed for a multi-layer backfill assuming that the layers were of uniform height and that the soil properties were also uniform over the wall height. To simplify the presentation the wall was assumed to be vertical and the backfill assumed to be cohesionless with a horizontal surface without superimposed loading.

Equations for N multiple-layers were derived using an extension of the Prakash and Basavanna method described above. Equations for the Stage 1 and 2 forces acting on the wall and their corresponding heights of action are:

$$P_{n1d} = \frac{(2n-1)k_h \cot \alpha}{[\cos \delta + \tan(\alpha - \varphi) \sin \delta]} \quad (42)$$

$$h_{n1d} = \frac{\frac{[3(n-1)+2]}{(2n-1)} [\cot \delta + \tan(\alpha - \varphi)] - 3(n-1) \cot \alpha}{[\cot \delta + \cot \alpha + \tan(\alpha - \varphi)]} \quad (43)$$

$$P_{n2d} = \frac{[2(N-n)+1] \cot \alpha}{[\sin \delta + \cot(\alpha - \varphi) \cos \delta]} \quad (44)$$

$$h_{n2d} = \frac{[6(N-n)(n-0.5) + 3(n-1) + 1] [\tan \delta + \cot(\alpha - \varphi)]}{[2(N-n) + 1] \cot(\alpha - \varphi)} - 3(n-1) \quad (45)$$

In Equations 42 to 45 the factor n denotes the layer number in the sequence with n for the top layer equal to the total number of layers, N . Each successive lower layer has an n value reducing by one.

3.1 Prakash and Basavanna Pressure Distribution

Direct application of the Prakash and Basavanna method gives the force acting on the wall and its height of application but not the shape of the pressure distribution. Prakash and Basavanna suggested that the active earth pressure $p(h)$ at a depth of h from the top of the wall could be expressed as:

$$p(h) = Kh^n \quad (46)$$

Where K and n are constants.

Using Equation 46 for the assumed pressure distribution on the wall the total force on the wall P_t and moment about the top of the wall M_t can be calculated from:

$$P_t = \int_0^H Kh^n dh = \frac{KH^{n+1}}{n+1} \quad (47)$$

$$M_t = \int_0^H Kh^{n+1} dh = \frac{KH^{n+2}}{n+2} \quad (48)$$

Dividing Equation 48 by Equation 47 and simplifying gives:

$$n = \frac{2M_t - P_t H}{P_t H - M_t} \quad (49)$$

The Prakash and Basavanna pressure distribution equation was investigated for a typical vertical wall having a height of 5.0 m and retaining a cohesionless backfill with a horizontal surface, no superimposed load, an internal friction angle of 35° , a wall-soil interface friction angle of 17.5° and a unit weight of 18 kN/m^3 . The horizontal earthquake acceleration was taken as $0.25 g$.

Evaluating Equations 49 and 47 for the calculated force and moment gave $n = 0.242$ and $K = 16.06 \text{ kN/m}^2$.

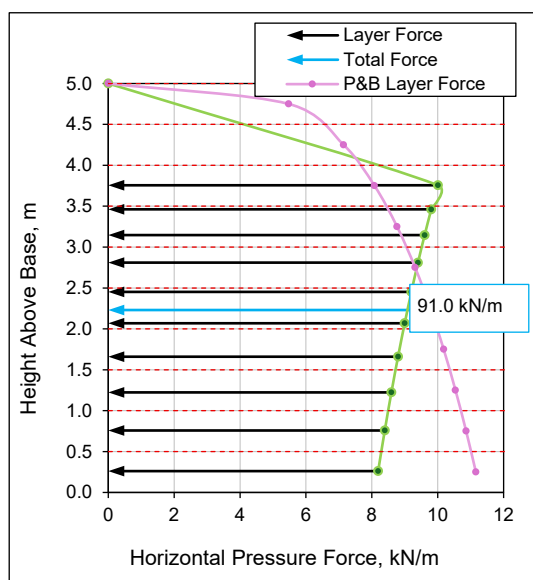


Figure 6 Pressure distributions for typical wall example.

4 CONCLUSIONS

A refined limit-state analysis for the seismic design of retaining walls that rigorously satisfies force and moment equilibrium of the soil failure wedge has been presented. The analysis is based on a reformulation and extension of the Prakash and Basavanna method and is compatible with generalized M-O formulations for total thrust magnitude.

The principal conclusions are as follows:

1. The commonly assumed linear pressure distribution with a resultant acting at one-third of the wall height does not in general satisfy moment equilibrium of the failure wedge and can be significantly unconservative for overturning and bearing capacity estimates.
2. The location of the resultant earth pressure force depends strongly on wall–soil interface friction, backfill geometry, seismic acceleration level, and the presence of cohesion or surcharge loading, and cannot be represented by a single fixed height ratio.
3. For smooth vertical walls under moderate seismic loading, the centre of pressure for combined gravity and earthquake load commonly exceeds $0.45H$, while for walls with low interface friction and sloping backfills it may exceed $0.7H$.
4. A multi-layer formulation allows approximation of the pressure distribution over the wall height and in many cases provides a more realistic representation of the loading on the wall than the pressure distribution formula presented by Prakash and Basavanna.

For routine designs involving low walls, low to moderate seismic loading, and significant wall interface friction, the conventional assumption of a resultant thrust acting at approximately $0.33H$ may be acceptable. However, for walls that retain sloping backfills, have low interface friction, or are located on sites subject to strong ground shaking and where wall overturning or soil bearing capacity governs use of the moment equilibrium-based method presented in this paper is recommended.

The results from the parametric studies and examples presented in the paper provide a rational means of identifying cases where simplified assumptions become unconservative and where additional refinement is justified.

The pressure distribution calculated using these factors in Equation 46 is shown in Figure 6. For comparison the distribution from a 10-layer uniform backfill calculated using Equations 42 to 45 is also shown. The pressure forces plotted are the horizontal components converted from the total pressure forces using the cosine of the wall-soil interface friction angle of 17.5° (a reduction of about 5%) and are the forces from the pressures over each layer of 0.5 m. For this typical low to moderate height wall, the Prakash and Basavanna distribution is not a good approximation to the more exact 10-layer analysis. Agreement is better for lower values of wall friction and earthquake accelerations.

If the pressure distribution is required for a wall with high wall-soil interface friction and/or high horizontal earthquake accelerations, then a multi-layer analysis should be used.

5 REFERENCES

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